MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER					

MATHEMATICS

9758/01

Paper 1

October/November 2019

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **26** printed pages and **2** blank pages.

Singapore Examinations and Assessment Board

CAMBRIDGE
International Examinations

JC19 11_9758_01/4R

[Turn over

1 The function f is defined by $f(z) = az^3 + bz^2 + cz + d$, where a, b, c and d are real numbers. Given that $2 + \mathrm{i}$ and -3 are roots of $f(z) = 0$, find b, c and d in terms of a. [4]

2 The curve C has equation $y = x^3 + x - 1$.

(i) C crosses the x-axis at the point with coordinates $(a, 0)$. Find the value of a correct to 3 decimal places. [1]

(ii) You are given that $b > a$.

The region P is bounded by C, the x-axis and the lines $x = -1$ and $x = 0$. The region Q is bounded by C, the line $x = b$ and the part of the x-axis between $x = a$ and $x = b$. Given that the area of Q is 2 times the area of P, find the value of b correct to 3 decimal places. [4]

3 A function is defined as $f(x) = 2x^3 - 6x^2 + 6x - 12$.

(i) Show that $f(x)$ can be written in the form $p\{(x + q)^3 + r\}$, where p, q and r are constants to be found. [2]

(ii) Hence, or otherwise, describe a sequence of transformations that transform the graph of $y = x^3$ onto the graph of $y = f(x)$. [3]

4 (i) Sketch the graph of $y = |2^x - 10|$, giving the exact values of any points where the curve meets the axes. [3]

(ii) Without using a calculator, and showing all your working, find the exact interval, or intervals, for which $|2^x - 10| \leqslant 6$. Give your answer in its simplest form. [3]

5 The functions f and g are defined by

$$f(x) = e^{2x} - 4, \quad x \in \mathbb{R},$$
$$g(x) = x + 2, \quad x \in \mathbb{R}.$$

(i) Find $f^{-1}(x)$ and state its domain. [3]

(ii) Find the exact solution of $fg(x) = 5$, giving your answer in its simplest form. [3]

6 (i) By writing $\dfrac{1}{4r^2 - 1}$ in partial fractions, find an expression for $\displaystyle\sum_{r=1}^{n} \frac{1}{4r^2 - 1}$. [4]

(ii) Hence find the exact value of $\displaystyle\sum_{r=11}^{\infty} \frac{1}{4r^2 - 1}$. [2]

7 A curve C has equation $y = xe^{-x}$.

(i) Find the equations of the tangents to C at the points where $x = 1$ and $x = -1$. [6]

(ii) Find the acute angle between these tangents. [2]

8 (a) An arithmetic series has first term a and common difference $2a$, where $a \neq 0$. A geometric series has first term a and common ratio 2. The kth term of the geometric series is equal to the sum of the first 64 terms of the arithmetic series. Find the value of k. [3]

(b) A geometric series has first term f and common ratio r, where $f, r \in \mathbb{R}$ and $f \neq 0$. The sum of the first four terms of the series is 0. Find the possible values of f and r. Find also, in terms of f, the possible values of the sum of the first n terms of the series. [4]

(c) The first term of an arithmetic series is negative. The sum of the first four terms of the series is 14 and the product of the first four terms of the series is 0. Find the 11th term of the series. [4]

9 **(i)** The complex number w can be expressed as $\cos\theta + \mathrm{i}\sin\theta$.

(a) Show that $w + \dfrac{1}{w}$ is a real number. [2]

(b) Show that $\dfrac{w-1}{w+1}$ can be expressed as $k\tan\frac{1}{2}\theta$, where k is a complex number to be found. [4]

(ii) The complex number z has modulus 1. Find the modulus of the complex number $\dfrac{z-3\mathrm{i}}{1+3\mathrm{i}z}$. [5]

10 A curve C has parametric equations

$$x = a(2\cos\theta - \cos 2\theta),$$
$$y = a(2\sin\theta - \sin 2\theta),$$

for $0 \leqslant \theta \leqslant 2\pi$.

(i) Sketch C and state the Cartesian equation of its line of symmetry. [2]

(ii) Find the values of θ at the points where C meets the x-axis. [2]

(iii) Show that the area enclosed by the x-axis, and the part of C above the x-axis, is given by

$$\int_{\theta_1}^{\theta_2} a^2(4\sin^2\theta - 6\sin\theta\sin 2\theta + 2\sin^2 2\theta)\,\mathrm{d}\theta,$$

where θ_1 and θ_2 should be stated. [3]

(iv) Hence find, in terms of a, the exact total area enclosed by C. [5]

11 Scientists are investigating how the temperature of water changes in various environments.

(i) The scientists begin by investigating how hot water cools.

The water is heated in a container and then placed in a room which is kept at a constant temperature of 16 °C. The temperature of the water t minutes after it is placed in the room is θ °C. This temperature decreases at a rate proportional to the difference between the temperature of the water and the temperature of the room. The temperature of the water falls from a value of 80 °C to 32 °C in the first 30 minutes.

(a) Write down a differential equation for this situation. Solve this differential equation to get θ as an exact function of t. [6]

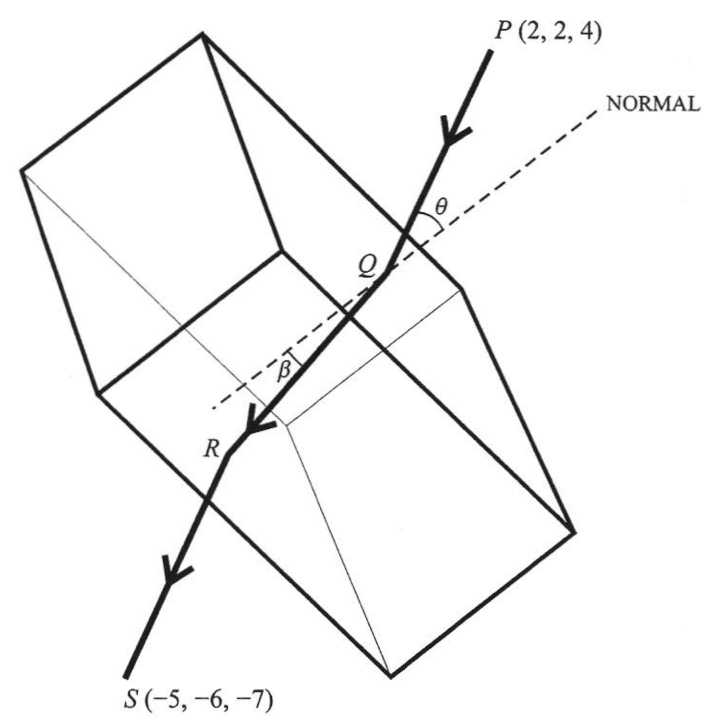
(b) Find the temperature of the water 45 minutes after it is placed in the room. [1]

(ii) The scientists then model the thickness of ice on a pond.

In winter the surface of the water in the pond freezes. Once the thickness of the ice reaches 3 cm, it is safe to skate on the ice. The thickness of the ice is T cm, t minutes after the water starts to freeze. The freezing of the water is modelled by a differential equation in which the rate of change of the thickness of the ice is inversely proportional to its thickness. It is given that $T = 0$ when $t = 0$. After 60 minutes, the ice is 1 cm thick.

Find the time from when freezing commences until the ice is first safe to skate on. [6]

12



A ray of light passes from air into a material made into a rectangular prism. The ray of light is sent in direction $\begin{pmatrix} -2 \\ -3 \\ -6 \end{pmatrix}$ from a light source at the point P with coordinates $(2, 2, 4)$. The prism is placed so that the ray of light passes through the prism, entering at the point Q and emerging at the point R and is picked up by a sensor at point S with coordinates $(-5, -6, -7)$. The acute angle between PQ and the normal to the top of the prism at Q is θ and the acute angle between QR and the same normal is β (see diagram).

It is given that the top of the prism is a part of the plane $x + y + z = 1$, and that the base of the prism is a part of the plane $x + y + z = -9$. It is also given that the ray of light along PQ is parallel to the ray of light along RS so that P, Q, R and S lie in the same plane.

(i) Find the exact coordinates of Q and R. [5]

(ii) Find the values of $\cos\theta$ and $\cos\beta$. [3]

(iii) Find the thickness of the prism measured in the direction of the normal at Q. [3]

Snell's law states that $\sin\theta = k\sin\beta$, where k is a constant called the refractive index.

(iv) Find k for the material of this prism. [1]

(v) What can be said about the value of k for a material for which $\beta > \theta$? [1]

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MATHEMATICS **9758/02**

Paper 2 **October/November 2019**

3 hours

Candidates answer on the Question Paper.

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JC19 11_9758_02/3R

Section A: Pure Mathematics [40 marks]

1 You are given that $I = \int x(1-x)^{\frac{1}{2}}\,dx$.

(i) Use integration by parts to find an expression for I. [2]

(ii) Use the substitution $u^2 = 1 - x$ to find another expression for I. [2]

(iii) Show algebraically that your answers to parts **(i)** and **(ii)** differ by a constant. [2]

2 **(i)** Sketch the graph of $y = \dfrac{2-x}{3x^2+5x-8}$. Give the equations of the asymptotes and the coordinates of the point(s) where the curve crosses either axis. [4]

(ii) Solve the inequality $\dfrac{2-x}{3x^2+5x-8} > 0$. [1]

(iii) Hence solve the inequality $\dfrac{x-2}{3x^2+5x-8} > 0$. [1]

3 A solid cylinder has radius r cm, height h cm and total surface area $900\,\text{cm}^2$. Find the exact value of the maximum possible volume of the cylinder. Find also the ratio $r : h$ that gives this maximum volume. [7]

4 **(i)** Given that $f(x) = \sec 2x$, find $f'(x)$ and $f''(x)$. Hence, or otherwise, find the Maclaurin series for $f(x)$, up to and including the term in x^2. [5]

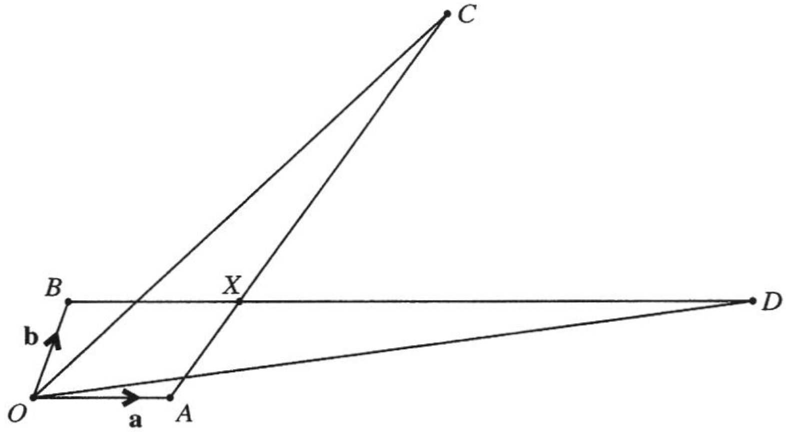
(ii) Use your series from part **(i)** to estimate $\int_0^{0.02} \sec 2x\,dx$, correct to 5 decimal places. [2]

(iii) Use your calculator to find $\int_0^{0.02} \sec 2x\,dx$, correct to 5 decimal places. [1]

(iv) Comparing your answers to parts **(ii)** and **(iii)**, and with reference to the value of x, comment on the accuracy of your approximations. [2]

(v) Explain why a Maclaurin series for $g(x) = \operatorname{cosec} 2x$ cannot be found. [1]

5



With reference to the origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = 2\mathbf{a} + 4\mathbf{b}$ and $\overrightarrow{OD} = \mathbf{b} + 5\mathbf{a}$. The lines BD and AC cross at X (see diagram).

(i) Express $\overrightarrow{OX}$ in terms of **a** and **b**. [4]

The point Y lies on CD and is such that the points O, X and Y are collinear.

(ii) Express $\overrightarrow{OY}$ in terms of **a** and **b** and find the ratio $OX : OY$. [6]

Section B: Probability and Statistics [60 marks]

6 In a certain country there are 100 professional football clubs, arranged in 4 divisions. There are 22 clubs in Division One, 24 in Division Two, 26 in Division Three and 28 in Division Four.

(i) Alice wishes to find out about approaches to training by clubs in Division One, so she sends a questionnaire to the 22 clubs in Division One. Explain whether these 22 clubs form a sample or a population. [1]

(ii) Dilip wishes to investigate the facilities for supporters at the football clubs, but does not want to obtain the detailed information necessary from all 100 clubs. Explain how he should carry out his investigation, and why he should do the investigation in this way. [2]

(iii) Find the number of different possible samples of 20 football clubs, with 5 clubs chosen from each division. [3]

7 A company produces drinking mugs. It is known that, on average, 8% of the mugs are faulty. Each day the quality manager collects 50 of the mugs at random and checks them; the number of faulty mugs found is the random variable F.

(i) State, in the context of the question, two assumptions needed to model F by a binomial distribution. [2]

You are now given that F can be modelled by a binomial distribution.

(ii) Find the probability that, on a randomly chosen day, at least 7 faulty mugs are found. [2]

(iii) The number of faulty mugs produced each day is independent of other days. Find the probability that, in a randomly chosen working week of 5 days, at least 7 faulty mugs are found on no more than 2 days. [2]

The company also makes saucers. The number of faulty saucers also follows a binomial distribution. The probability that a saucer is faulty is p. Faults on saucers are independent of faults on mugs.

(iv) Write down an expression in terms of p for the probability that, in a random sample of 10 saucers, exactly 2 are faulty. [1]

The mugs and saucers are sold in sets of 2 randomly chosen mugs and 2 randomly chosen saucers. The probability that a set contains at most 1 faulty item is 0.97.

(v) Write down an equation satisfied by p. Hence find the value of p. [4]

8 Gerri collects characters given away in packets of breakfast cereal. There are four different characters: Horse, Rider, Dog and Bird. Each character is made in four different colours: Orange, Yellow, Green and White. Gerri has collected 56 items; the numbers of each character and colour are shown in the table.

	Orange	Yellow	Green	White
Horse	1	1	3	4
Rider	1	1	7	5
Dog	3	7	1	6
Bird	4	5	6	1

(i) Gerri puts all the items in a bag and chooses one item at random.

(a) Find the probability that this item is either a Horse or a Rider. [1]

(b) Find the probability that this item is either a Dog or a Bird but the item is not White. [1]

(ii) Gerri now puts the item back in the bag and chooses two items at random.

(a) Find the probability that both of the items are Horses, but neither of the items is Orange. [1]

(b) Find the probability that Gerri's two items include exactly one Dog and exactly one item that is Yellow. [3]

(iii) Gerri has two favourites among the 16 possible colour/character combinations. The probability of choosing these two at random from the 56 items is $\frac{1}{77}$. Write down all the possibilities for Gerri's two favourite colour/character combinations. [3]

9 A company produces resistors rated at 750 ohms for use in electronic circuits. The production manager wishes to test whether the mean resistance of these resistors is in fact 750 ohms. He knows that the resistances are normally distributed with variance 100 ohms^2.

(i) Explain whether the manager should carry out a 1-tail test or a 2-tail test. State hypotheses for the test, defining any symbols you use. [2]

The production manager takes a random sample of 8 of these resistors. He finds that the resistances, in ohms, are as follows.

742 771 768 738 769 752 742 766

(ii) Find the mean of the sample of 8 resistors. Carry out the test, at the 5% level of significance, for the production manager. Give your conclusion in context. [5]

The company also produces resistors rated at 1250 ohms. Nothing is known about the distribution of the resistances of these resistors.

(iii) Describe how, and why, a test of the mean resistance of the 1250 ohms resistors would need to differ from that for the 750 ohms resistors. [2]

10 Abi and Bhani find the fuel consumption for a car driven at different constant speeds. The table shows the fuel consumption, y kilometres per litre, for different constant speeds, x kilometres per hour.

x	40	45	50	55	60
y	22	20	18	17	16

(i) Abi decides to model the data using the line $y = 35 - \frac{1}{3}x$.

(a) On the grid

- draw a scatter diagram of the data,
- draw the line $y = 35 - \frac{1}{3}x$. [2]

(b) For a line of best fit $y = \mathrm{f}(x)$, the residual for a point (a, b) plotted on the scatter diagram is the vertical distance between $(a, \mathrm{f}(a))$ and (a, b). Mark the residual for each point on your diagram. [1]

(c) Calculate the sum of the squares of the residuals for Abi's line. [1]

(d) Explain why, in general, the sum of the squares of the residuals rather than the sum of the residuals is used. [1]

Bhani models the same data using a straight line passing through the points (40, 22) and (55, 17). The sum of the squares of the residuals for Bhani's line is 1.

(ii) State, with a reason, which of the two models, Abi's or Bhani's, gives a better fit. [1]

(iii) State the coordinates of the point that the least squares regression line must pass through. [1]

(iv) Use your calculator to find the equation of the least squares regression line of y on x. State the value of the product moment correlation coefficient. [3]

(v) Use the equation of the regression line to estimate the fuel consumption when the speed is 30 kilometres per hour. Explain whether you would expect this value to be reliable. [2]

(vi) Cerie performs a similar experiment on a different car. She finds that the sum of the squares of the residuals for her line is 0. What can you deduce about the data points in Cerie's experiment? [1]

11 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

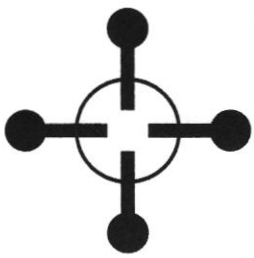
Arif is making models of hydrocarbon molecules. Hydrocarbons are chemical compounds made from carbon atoms and hydrogen atoms. Arif has a bag containing a large number of white balls to represent the carbon atoms, and a bag containing a large number of black balls to represent the hydrogen atoms. The masses of the white balls have the distribution $\mathrm{N}(110, 4^2)$ and the masses of the black balls have the distribution $\mathrm{N}(55, 2^2)$. The units for mass are grams.

(i) Find the probability that the total mass of 4 randomly chosen white balls is more than 425 grams. [2]

(ii) Find the probability that the total mass of a randomly chosen white ball and a randomly chosen black ball is between 161 and 175 grams. [2]

(iii) The probability that 2 randomly chosen white balls and 3 randomly chosen black balls have total mass less than M grams is 0.271. Find the value of M. [4]

Arif also has a bag containing a large number of connecting rods to fix the balls together. The masses of the connecting rods, in grams, have the distribution $N(20, 0.9^2)$. In order to make models of methane (a hydrocarbon), Arif has to drill 1 hole in each black ball, and 4 holes in each white ball, for the connecting rods to fit in. This reduces the mass of each black ball by 10% and reduces the mass of each white ball by 30%.



A methane molecule consists of 1 carbon atom and 4 hydrogen atoms. Arif makes a model of a methane molecule using 4 black balls, 1 white ball and 4 connecting rods (see diagram). The balls and connecting rods are all chosen at random.

(iv) Find the probability that the mass of Arif's model is more than 350 grams. [4]