ANDERSON SERANGOON JUNIOR COLLEGE

MATHEMATICS **9758**

H2 Mathematics Paper 2 (100 marks)

18 Sept 2020

3 hours

Additional Material(s): List of Formulae (MF 26)

CANDIDATE NAME

CLASS /

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
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10	
Total	

This document consists of 18 printed pages and 2 blank pages.

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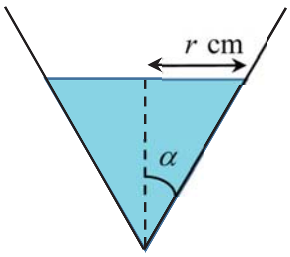
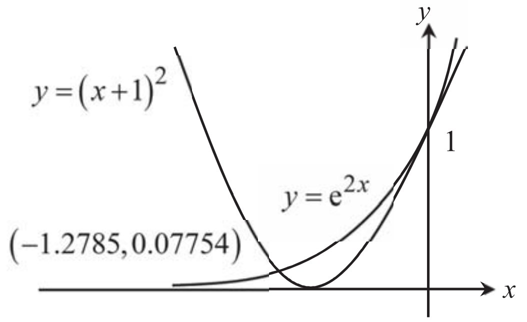
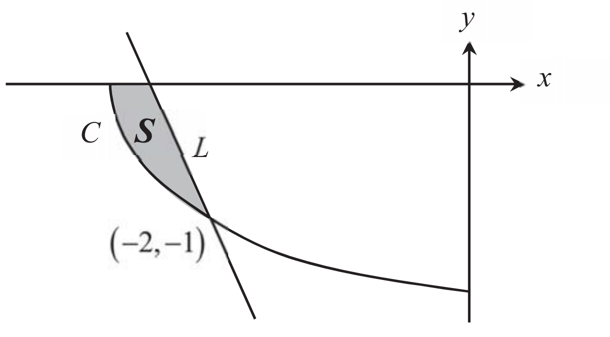
	Section A: Pure Mathematics [40 marks]	
1	The complex number z and w are given by $z=2\mathrm{e}^{\mathrm{i}\theta}$, where $\frac{3\pi}{4}<\theta\le\pi$, and $w=1+\sqrt{3}\mathrm{i}$. Express $w-z$ in exponential form $r\mathrm{e}^{i\alpha}$, where both r and α are in terms of θ and $-\pi<\alpha\le\pi$.	[3]
2	**(a)** A curve is defined by the parametric equations $$x=\frac{1}{t},\qquad y=t^2,\qquad \text{for } 0<t<1.$$ Show that the equation of the normal to the curve at the point $P\left(\frac{1}{p},p^2\right)$ is $$2p^4y-px=2p^6-1$$ Hence, using an algebraic method, show that the normal at P cuts the curve exactly once.	[6]
	(b) 	
	A cone of semi-vertical angle α, where $\tan\alpha=\frac{1}{3}$, is held with its axis vertical and vertex downwards (see diagram). At the beginning of an experiment, it is filled with 300 cm^3 of liquid water. Water runs out of a small hole at the vertex at a constant rate of 0.2 cm^3 per second. At time t minutes after the start, the radius of the water surface is r cm (see diagram). Find the rate at which the depth of the liquid is decreasing 5 minutes after the start of the experiment. [The volume of a cone of base radius r and height h is given by $V=\frac{1}{3}\pi r^2h$.]	[5]
3	A drone carrying a bomb departs from a point $A(-1,1,-3)$. It flies in a direction of $-a\mathbf{i}+\mathbf{j}+2a\mathbf{k}$, where $a>0$, across a lake to a target location. It is given that the surface of the lake is part of a plane with equation $x-z=2$.	
	(i) Determine the value of a if the path of the drone makes an angle of $\frac{\pi}{3}$ radians with the surface of the lake.	[3]
	It is given that $a=2$.	
	(ii) A missile is launched to intercept the drone. It moves in a path with equation $\frac{x-10}{2}=z,\ y=m$, where m is a real constant. Given that the missile is successful in intercepting the drone, find the point of interception.	[3]

	(iii) At the point of interception, a piece of the drone falls perpendicular to the lake and meets the surface of the lake at N. Find the position vector of N.	[3]
	(iv) Denoting the line that contains the drone's path as L_1 and $L_1{}'$ being the reflection of L_1 in the surface of the lake, find a vector equation of $L_1{}'$.	[3]
4	**(a)** Find $\int \left(\cot^6 2x + \cot^4 2x - \sin\frac{x}{2}\sin\frac{3x}{2} \right) \mathrm{d}x$.	[3]
	(b) The diagram shows the graphs of $y = \mathrm{e}^{2x}$ and $y = (x+1)^2$.	
		
	R is the finite region bounded by the two curves $y = \mathrm{e}^{2x}$ and $y = (x+1)^2$. Find the volume of the solid formed when R is rotated through 2π radians about the y-axis, giving your answer correct to 4 decimal places.	[3]
	(c) A curve C has parametric equations $$x = 2\left(\sin\frac{t}{2} - \cos\frac{t}{2}\right), \quad y = \sin\frac{t}{2} + \cos\frac{t}{2}, \quad \text{for } -\frac{3\pi}{2} \le t \le -\frac{\pi}{2}.$$	
	The line L with equation $y = -2x - 5$ meets the curve C at the point $(-2, -1)$. S is the region enclosed by line L, curve C and the x-axis as shown in the diagram below.	
		
	(i) Show that the x-intercept of curve C is $-2\sqrt{2}$.	[2]
	(ii) Find the exact area of S.	[6]
	Section B: Statistics [60 marks]	
5	Elaine always receives two \$2 notes, two \$5 notes and one \$20 note from her parents as her monthly allowance. She decides to select two of these notes to contribute to Community Chest for the month of August. The total value of these notes is denoted by \$$C$.	
	(i) Determine the probability distribution of C.	[3]

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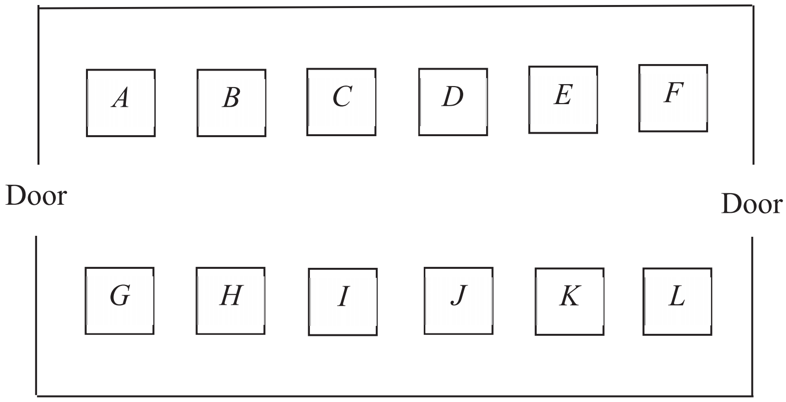
	(ii) Find the expected amount of money that she will donate in August and the variance of C.	[2]
	In September, Elaine decides to contribute to Community Chest again.	
	(iii) Find the probability that her total amount of donations for August and September is at least \$12.	[2]
6	A group of 300 students are asked whether they own any earbuds, laptops or games machines. 90 students own a pair of earbuds 177 students own a laptop 100 students own a games machine	
	Events A, L and G are defined as follows:	
	A: a randomly chosen student owns a pair of earbuds. L: a randomly chosen student owns a laptop. G: a randomly chosen student owns a game machine.	
	It is given that events A and G are independent events. It is also given that 35 students own a laptop and a games machine and that 20 students own all the three gadgets.	
	(i) Find the probability that a student selected at random owns either a games machine or a pair of earbuds but not both.	[2]
	(ii) A student selected at random owns a games machine. Find the probability that the student owns exactly two of the gadgets.	[2]
	(iii) Find the greatest and least value of $\mathrm{P}\left(A \cap L \cap G'\right)$.	[3]
7	The masses, in grams, of mangoes have the distribution $\mathrm{N}(200, 30^2)$ and the masses, in grams, of oranges have the distribution $\mathrm{N}(\mu, \sigma^2)$.	
	(i) Let A be the average mass of a mango and two oranges. Given that $\mathrm{P}(A < 162) = \mathrm{P}(A > 206) = 0.06$, find μ. Hence form an equation involving σ and solve for σ.	[5]
	(ii) Find probability that the total mass of 3 mangoes differs from 3 times the mass of an orange by at most 136 grams.	[3]
	(iii) Mangoes are sold at \$26 per kilogram. Find the probability that the total selling price of 6 mangoes exceeds \$29.50.	[2]
8	The boss of a hair salon chain claims that the waiting time, on average, is 15 minutes. Jeremy believes that the waiting time quoted by the hair salon chain is understated. He carries out a survey to investigate this by asking the waiting time for 110 customers. The waiting time are summarised by	
	$\sum t = 1900$, and $\sum (t-15)^2 = 25400$.	
	(i) Find the unbiased estimate of the population mean. Leave your answers correct to 3 decimal places. Show that the unbiased estimate of the population variance is given by 227.815.	[2]
	(ii) Carry out a test on Jeremy's belief at the 5% significance level, stating a necessary assumption for the test.	[5]
	(iii) Kalai did another test to determine whether the mean waiting time differs from 15 minutes using another random sample. The waiting times of n	[4]

	randomly chosen customers, where n is large, were recorded, and their mean and standard deviation are 15.6 minutes and 2.5 minutes respectively. Given that the null hypothesis is rejected at the 5% level of significance, find the least value of n.	
9	A rectangular barrack with a door at each end, has 12 beds marked A, B, C,..., L as shown in the diagram below. Johnson and 11 other soldiers take 1 bed each.	
		
	(i) Find the number of different sleeping arrangements possible if none of Johnson and 2 other particular soldiers are adjacent to each other, and all three of them are on the same side of the barrack.	[2]
	One afternoon, 9 particular soldiers went out to have lunch. There were 3 identical round tables at the dining venue.	
	(ii) How many ways can 9 soldiers be seated if there must be at least two soldiers at each table?	[4]
	Johnson was playing with a deck of 10 cards. When the cards are arranged in a certain manner, it forms the word SUCCESSFUL.	
	(iii) Find the number of ways he can arrange the 10 cards such that not all the letter "S" are together.	[2]
	(iv) He wishes to replace his existing handphone 4-letter codeword using letters from the word SUCCESSFUL, how many different options would he have.	[4]
10	A ferry company operates a 9am ferry, with a passenger capacity of 108, from Island X to Island Y daily. To maximise profits, 113 tickets are sold online each day because it was found that on average, p% of customers who have purchased a ticket do not turn up.	
	(i) It is known that there is a probability of 0.012 that at most 1 customer will not turn up for the ferry. Write down an equation in terms of p, and hence find p correct to 3 decimal places.	[3]
	It is now given that $p = 6$.	
	(ii) Find the probability that there are no empty seats if every customer who turns up get a seat on the 9am ferry.	[3]
	(iii) Find the probability that, in a randomly chosen week, every customer who turns up gets a seat on the 9am ferry on at least 5 days of the week.	[2]
	(iv) Find the maximum number of tickets that could be sold so that the probability of a customer not getting a seat on the ferry is no more than 0.01.	[3]

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	(v) A period of forty days are randomly selected. Find the probability that the mean daily number of customers who purchased a ticket and managed to turn up during this period is at most 106.	[2]

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