	Section A: Pure Mathematics [40 marks]	
1	The complex number z and w are given by $z = 2e^{i\theta}$, where $\dfrac{3\pi}{4} < \theta \le \pi$, and $w = 1 + \sqrt{3}i$. Express $w - z$ in exponential form $re^{i\alpha}$, where both r and α are in terms of θ and $-\pi < \alpha \le \pi$.	[3]
	Solution	
	Express as $w = 2e^{i\frac{\pi}{3}}$.	
	$w - z = 2e^{i\frac{\pi}{3}} - 2e^{i\theta}$	
	$= 2e^{i\left(\frac{\pi+3\theta}{6}\right)}\left(e^{i\left(\frac{\pi-3\theta}{6}\right)} - e^{-i\left(\frac{\pi-3\theta}{6}\right)}\right)$	
	$= 2e^{i\left(\frac{3\theta+\pi}{6}\right)} \times 2i \times \underbrace{\sin\left(\dfrac{\pi-3\theta}{6}\right)}_{\text{negative}}$ $\left[\dfrac{\pi-3\theta}{6} \text{ is in the 4}^{\text{th}} \text{ quadrant}\right]$	
	$= 2e^{i\left(\frac{3\theta+\pi}{6}\right)} \times 2i \times (-1)\underbrace{\sin\left(\dfrac{3\theta-\pi}{6}\right)}_{\text{positive}}$ $[\text{since } \sin(-x) = -\sin x]$	
	$= 4\sin\left(\dfrac{3\theta-\pi}{6}\right)e^{i\left(\frac{3\theta+\pi}{6}\right)}e^{i\left(-\frac{\pi}{2}\right)}$ $[\text{since } -i = e^{i\left(-\frac{\pi}{2}\right)}]$	
	$= 4\sin\left(\dfrac{\theta}{2} - \dfrac{\pi}{6}\right)e^{i\left(\frac{\theta}{2} - \frac{\pi}{3}\right)}$	
2	**(a)** A curve is defined by the parametric equations $$x = \frac{1}{t}, \qquad y = t^2, \qquad \text{for } 0 < t < 1.$$ Show that the equation of the normal to the curve at the point $P\left(\dfrac{1}{p}, p^2\right)$ is $$2p^4 y - px = 2p^6 - 1.$$ Hence, using an algebraic method, show that the normal at P cuts the curve exactly once.	[6]
	(b)	

Commented [LMH1]: Misconception
A worrying observation is that some students erroneously think that
|w + z| = |w| + |z|,
arg(w + z) = arg(w) + arg(z).
These are false! For these students, do make it a point to clarify with your tutors.
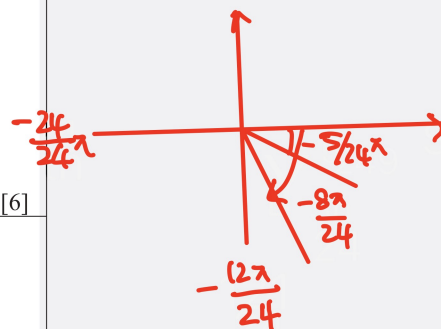
Commented [LMH2]: Things to note
Taking out "the average of the arguments" will help surface addition/subtraction of complex conjugates. And we know that this can then be simplified.

Commented [LMH3]: Things to note
For exponential form $re^{i\theta}$, the modulus r must be a **positive real number**.

A cone of semi-vertical angle α, where $\tan\alpha = \dfrac{1}{3}$, is held with its axis vertical and vertex downwards (see diagram). At the beginning of an experiment, it is filled with 300 cm³ of water. Water runs out of a small hole at the vertex at a constant rate of 0.2 cm³ per second. At time t minutes after the start, the radius of the water surface is r cm (see diagram). Find the rate at which the depth of the water is decreasing 5 minutes after the start of the experiment.

[The volume of a cone of base radius r and height h is given by $V = \dfrac{1}{3}\pi r^2 h$.] [5]

Solution

(a)

$x = \dfrac{1}{t}$ $\qquad y = t^2$

$\dfrac{dx}{dt} = -\dfrac{1}{t^2}$ $\qquad \dfrac{dy}{dt} = 2t$

$\dfrac{dy}{dx} = \dfrac{2t}{-\dfrac{1}{t^2}} = -2t^3$

Equation of normal at point P:

$y - p^2 = \dfrac{1}{2p^3}\left(x - \dfrac{1}{p}\right)$

$2p^4 y - 2p^6 = px - 1$

$2p^4 y - px = 2p^6 - 1$

To check if the normal cuts the curve again,

$2p^4\left(t^2\right) - p\left(\dfrac{1}{t}\right) = 2p^6 - 1$

[handwritten] $t = p$ $\left(\dfrac{1}{p}, p^2\right)$

$2p^4 t^3 - p + (1 - 2p^6)t = 0$

[handwritten] cubic function.

$(t - p)\left(2p^4 t^2 + At + 1\right) = 0$

[handwritten] quadratic $ax^2 + bx + c$

Comparing coefficient of t: $1 - Ap = 1 - 2p^6$

[handwritten] since $a = 2p^4$, $c = 1$ (By observation) then $b = A \times t$

$A = 2p^5$

$\therefore (t - p)\left(2p^4 t^2 + 2p^5 t + 1\right) = 0$

$t = p$ or $2p^4 t^2 + 2p^5 t + 1 = 0 \ldots (1)$

For $2p^4 t^2 + 2p^5 t + 1 = 0$,

Discriminant $= \left(2p^5\right)^2 - 4\left(2p^4\right)(1)$

$= 4p^{10} - 8p^4$

$= 4p^4\left(p^6 - 2\right) < 0$ since $0 < p < 1$ (Need to state the reason)

Commented [SH4]: <u>Things to note:</u>
This is a crucial step for students to check if there is any other solution if the normal and the curve were to intersect.

Commented [SH5]: <u>Things to Note:</u>
Since Point P can be the only solution, $t = p$ is the only root. Therefore, expressing $(t - p)\left(2p^4 t^2 + At + 1\right)$ is important.

Commented [SH6]: <u>Presentation</u>
Algebraic Method is only acceptable. Students should make it a point to show that, Discriminant $4p^{10} - 8p^4 < 0$ and state the reason why this is so. Since it's a show question.
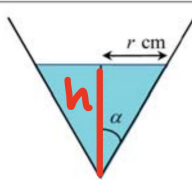
[Turn Over

[handwritten long division]

Long Division.

$2p^4 t^2 + 2p^5 t + 1$

$(t-p)\overline{\smash{\big)}\,2p^4 t^3 - p + (1 - 2p^6)t}$

$-\underline{(2p^4 t^3 - 2p^5 t^2)}$

$2p^5 t^2 - p + t - 2p^6 t$

$-\underline{(2p^5 t^2 - 2p^6 t)}$

$-p + t$

Hence (1) has no real solution.
Since there is only 1 real solution for t, so the normal at P cuts the curve only once.

Commented [KSM7]: Need to state the implication of Disc < 0 to fully answer why there's only 1 real root.

(b) Let V be the volume of the water in the cone and h be the depth of water in the cone. Then,

$$V = \frac{1}{3}\pi r^2 h$$

$$\tan\alpha = \frac{r}{h} = \frac{1}{3}$$

$$r = \frac{1}{3}h$$

$\tan\alpha = \frac{1}{3}$

$= \frac{r}{h} = \frac{1}{3}$

$\therefore 3r = h.$

$$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{27}\pi h^3 \qquad \frac{dv}{dh} =$$

Commented [KSM8]: __Things to note:__
Some attempted to use V in terms of r to solve, but often were unable to continue to get $\frac{dh}{dt}$, so lost the marks for linking to the depth. There were very few successful attempts though, who see that $\frac{dh}{dt} = 3\frac{dr}{dt}$.

Initial volume $= 300 \text{ cm}^3$
Decrease in volume after 5 min $= 0.2 \times 60 \times 5$
$= 60 \text{ cm}^3$
Volume of the water in the cone $= 240 \text{ cm}^3$

$$240 = \frac{1}{27}\pi h^3$$

$$h^3 = \frac{6480}{\pi}$$

$$h = \sqrt[3]{\frac{6480}{\pi}}$$

$$\frac{dV}{dh} = \frac{1}{9}\pi h^2$$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dh} = 3\left(\frac{1}{27}\right)\pi h^2$

$= \frac{1}{9}\pi h^2$

$$\frac{dV}{dh} = \frac{1}{9}\pi h^2$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$-0.2 = \frac{1}{9}\pi\left(\frac{6480}{\pi}\right)^{\frac{2}{3}}\frac{dh}{dt}$$

Commented [SH9]: __Careless Mistake:__
Many students have used $\frac{dV}{dt} = 0.2$ instead of $\frac{dV}{dt} = -0.2$

$$\frac{dh}{dt} = -0.00354 \text{ cm/s}$$

The height is decreasing at a rate of 0.00354 cm/s.

Commented [SH10]: __Presentation__
1. $\frac{dh}{dt} = -0.00354$ cm/s
2. Rate of decrease = 0.00354 cm/s. mean the same but said differently.

3 A drone carrying a bomb departs from a point $A(-1, 1, -3)$. It flies in a direction of $-a\mathbf{i} + \mathbf{j} + 2a\mathbf{k}$, where $a > 0$, across a lake to a target location. It is given that the surface of the lake is part of a plane with equation $x - z = 2$.
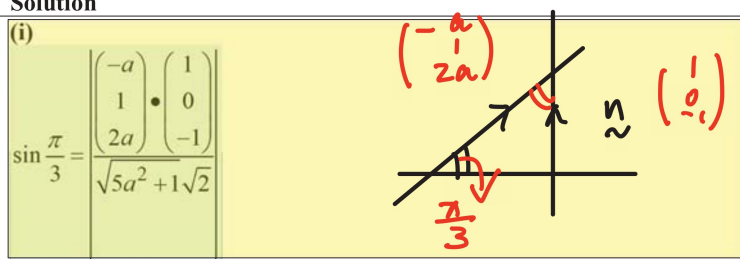
Commented [NCY11]: Question reading Students didn't realise this is the direction vector of the path and not a position vector of a point.

(i) Determine the value of a if the path of the drone makes an angle of $\frac{\pi}{3}$ radians with the surface of the lake. [3]

It is given that $a = 2$.

Commented [NCY12]: Question reading Students didn't realise this given condition is needed to reject the value of a

Commented [LT13]: __Question Reading__ A number did not realized that this is the angle between a line and a plane.

A
$(-1, 1, -3)$

$\begin{pmatrix} -a \\ 1 \\ 2a \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$
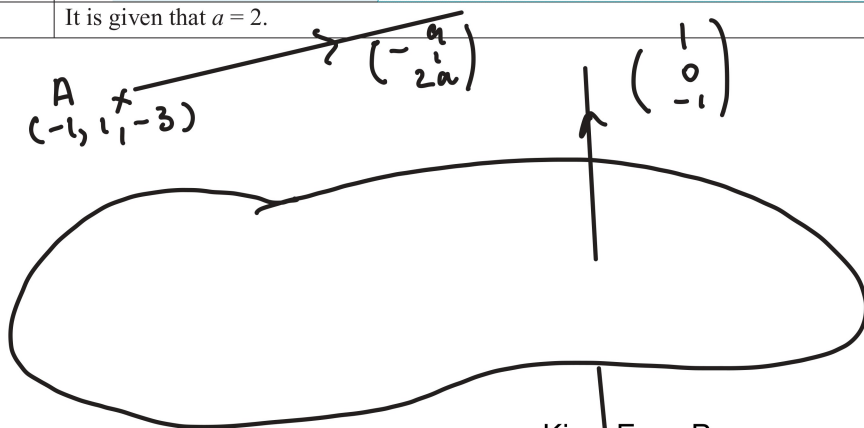
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ m \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$$

(ii) A missile is launched to intercept the drone. It moves in a path with equation $\dfrac{x-10}{2} = z, \ y = m$, where m is a real constant. Given that the missile is successful in intercepting the drone, find the point of interception. [3]	**Commented [LT14]:** Misconception A number of students could not convert this equation accurately in vector equation form.
(iii) At the point of interception, a piece of the drone falls perpendicular to the lake and meets the surface of the lake at N. Find the position vector of N. [3]	
(iv) Denoting the line that contains the drone's path as L_1 and L_1' being the reflection of L_1 in the surface of the lake, find a vector equation of L_1'. [3]	
Solution	
(i) $\sin \dfrac{\pi}{3} = \dfrac{\left| \begin{pmatrix} -a \\ 1 \\ 2a \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right|}{\sqrt{5a^2+1}\sqrt{2}}$ 	**Commented [LT15]:** Misconception A number of students failed to put a modulus on the right hand side of the equation. This is because $\sin \dfrac{\pi}{3}$ is positive whereas the dot product on the right hand side may give rise to a negative outcome if not careful.
$(6a)^2 = 6(5a^2+1)$	
$a = 1 \quad \text{or} \quad a = -1 \text{ (rejected since } a > 0)$	**Commented [LT16]:** Presentation of Answer Some did not explain why the value of a is 1 and not -1.
(ii) Given that $a = 2$. $l_{\text{drone}}: \ \mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}, \lambda \in \mathbb{R}$	
$l_{\text{missile}}: \ \mathbf{r} = \begin{pmatrix} 10 \\ m \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$ $\begin{pmatrix} -1-2\lambda \\ 1+\lambda \\ -3+4\lambda \end{pmatrix} = \begin{pmatrix} 10+2\mu \\ m \\ \mu \end{pmatrix}$	**Commented [NCY17]:** Misconception Many are unable to convert to the vector equation accurately
$-2\mu - 2\lambda = 11$ $-\mu + 4\lambda = 3$	
$\lambda = -\dfrac{1}{2}, \ \mu = -5$	**Commented [NCY18]:** Question reading As question ask for a point, the answer must be presented in a coordinate form As question didn't ask for the point in terms of m, thus it is expected the answer is a value. In any case, the value of m is not needed to solve for the point
$m = \dfrac{1}{2}$	
Coordinates of point of intersection are $\left(0, \dfrac{1}{2}, -5\right)$.	

[Turn Over

point of interception ↗

Equation of line ↘

(iii) Let the point of interception be P.

$\overrightarrow{ON}$ ← $\ell_{NP} : \mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ -5 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \ \alpha \in \mathbb{R}$ *for same*

Since N ~~also~~ lies on the plane

$\begin{pmatrix} \alpha \\ \frac{1}{2} \\ -5-\alpha \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 2$ for some $\alpha \in \mathbb{R}$

$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \underset{\sim}{\mathbf{n}}$ *plane.*

At N must lie on ℓ plane.

$\underset{\sim}{v} \cdot \underset{\sim}{n} = \underset{\sim}{a} \cdot \underset{\sim}{n} = 2$

$2\alpha = -3$

$\alpha = -\dfrac{3}{2}$

$\overrightarrow{ON} = \dfrac{1}{2}\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix}$

OR

$\overrightarrow{NP} = \left(\begin{pmatrix} 1 \\ -\frac{1}{2} \\ -2 \end{pmatrix} \cdot \dfrac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}} \right) \dfrac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}}$
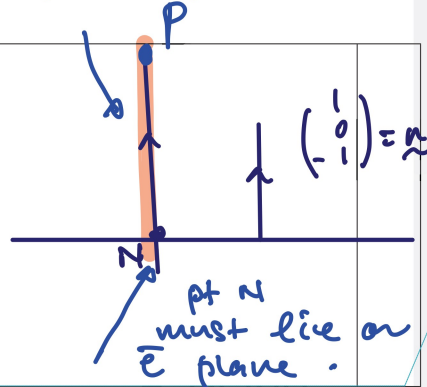
$= \dfrac{1}{2}\begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$

$\overrightarrow{ON} = \begin{pmatrix} 0 \\ 1 \\ -5 \end{pmatrix} - \dfrac{1}{2}\begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} = \dfrac{1}{2}\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix}$

(iv)
Let P' be the reflected point of P in the lake.

Using ratio theorem, $\overrightarrow{ON} = \dfrac{\overrightarrow{OP} + \overrightarrow{OP'}}{2}$

$\Rightarrow \overrightarrow{OP'} = 2\overrightarrow{ON} - \overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} - \dfrac{1}{2}\begin{pmatrix} 0 \\ 1 \\ -10 \end{pmatrix} = \begin{pmatrix} -3 \\ \frac{1}{2} \\ -2 \end{pmatrix}$

Commented [LT19]: Misconception
Wrote $\overrightarrow{PN} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 2$ or $\overrightarrow{AN} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 2$

Commented [LT20]: Misconception
Most who did using the projection vector approach often ends up getting the wrong answer because of a wrong formula used or a wrong direction of the vector quoted. It is highly recommended **NOT TO USE the projection vector method.**

Incorrect to write is as

$\overrightarrow{PN} = \left| \begin{pmatrix} 1 \\ -\frac{1}{2} \\ -2 \end{pmatrix} \cdot \dfrac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}} \right| \dfrac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{2}}$ because there is no

modulus in the formula when using the projection vector approach.
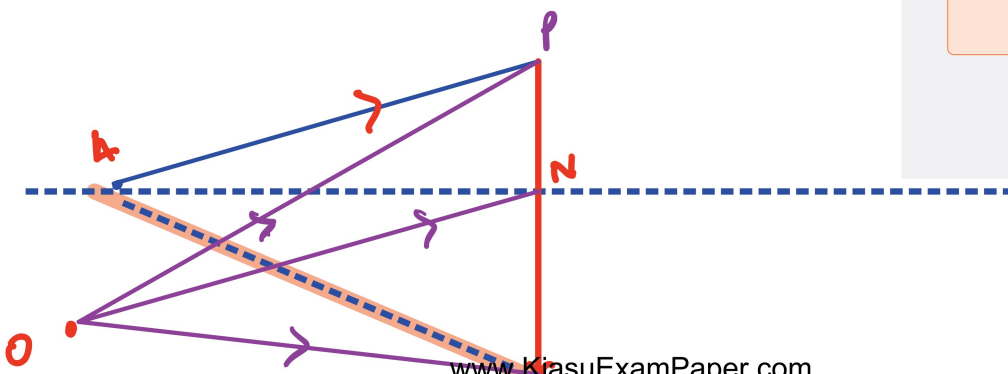
Commented [NCY21]: Misconception
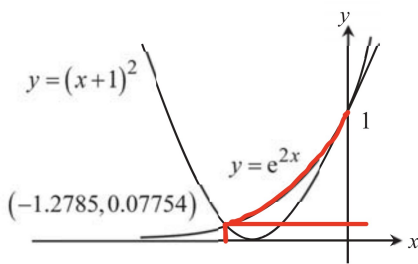Midpoint theorem needed to be stated accurately
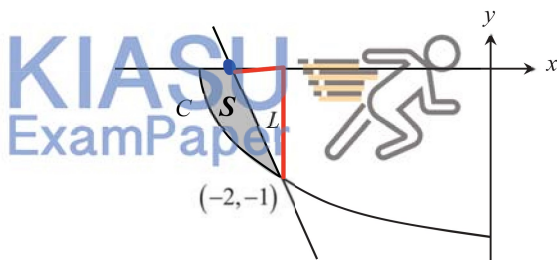
Misconception:
$\overrightarrow{ON} = \dfrac{\overrightarrow{AP} + \overrightarrow{AP'}}{2}$

$\overrightarrow{AN} = \dfrac{\mathbf{d}_L + \mathbf{d}_L}{2}$

$\overrightarrow{ON} = \dfrac{\overrightarrow{OA} + \overrightarrow{OA'}}{2}$

The direction vector of the reflected line is $\begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} -3 \\ \frac{1}{2} \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$

> **Commented [NCY22]:** Misconception
> Direction vector of the reflected line cannot be the same as the original line when they are not even parallel

$\therefore L_1' : \mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + \beta \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}, \beta \in \mathbb{R}$

> **Commented [NCY23]:** Presentations
> Presentation of direction vector should always be in simplest form

4 **(a)** Find $\displaystyle\int \left(\cot^6 2x + \cot^4 2x - \sin\frac{x}{2}\sin\frac{3x}{2} \right) dx$. [3]

(b) The diagram shows the graphs of $y = e^{2x}$ and $y = (x+1)^2$.

R is the finite region bounded by the two curves $y = e^{2x}$ and $y = (x+1)^2$.
Find the volume of the solid formed when R is rotated through 2π radians about the y-axis giving your answer correct to 4 decimal places. [3]

(c) A curve C has parametric equations

$$x = 2\left(\sin\frac{t}{2} - \cos\frac{t}{2} \right), \qquad y = \sin\frac{t}{2} + \cos\frac{t}{2}, \qquad \text{for } -\frac{3\pi}{2} \le t \le -\frac{\pi}{2}.$$

The line L with equation $y = -2x - 5$ meets the curve C at the point $(-2, -1)$.
S is the region enclosed by line L, curve C and the x-axis as shown in the diagram below.

(i) Show that the x-intercept of curve C is $-2\sqrt{2}$. [2]
(ii) Find the exact area of S. [6]

Solution

[handwritten right margin]
$\pi \int x^2 \, dy$

$y = e^{2x}$
$\ln y = 2x \ln e$
$\therefore x = \frac{1}{2}(\ln y)$

$y = (x+1)^2$
$\pm\sqrt{y} = x + 1$
$\therefore x = -1 \pm \sqrt{y}$

[handwritten bottom]
$P_1 + P_2 = \text{Total} \div 2$

[Turn Over

$$\int f'(x) \cdot f(x)^n \, dx = \frac{f(x)^{n+1}}{n+1} + C$$

$$\tan^2 x + 1 = \sec^2 x$$
$$\cot^2 x + 1 = \csc^2 x$$

Formula.
$$\therefore \frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

8

(a) $\displaystyle\int \left(\cot^6 2x + \cot^4 2x - \sin\frac{x}{2}\sin\frac{3x}{2} \right) dx$ ← Factor formula.

$$= \int \left(\cot^4 2x \left(\cot^2 2x + 1 \right) + \frac{1}{2}(\cos 2x - \cos x) \right) dx$$

$$= \int \left(\cot^4 2x \left(\csc^2 2x \right) + \frac{1}{2}(\cos 2x - \cos x) \right) dx$$

$f(x) = \cot 2x$
$f'(x) = -2\csc^2 2x$

$$= -\frac{\cot^5 2x}{10} + \frac{\sin 2x}{4} - \frac{\sin x}{2} + c$$

$\frac{d}{dx}(\cot 2x) = -2\csc^2 2x$

$-\frac{1}{2}\int -2\csc^2 x \cdot \cot^4 2x \, dx$

(b) Volume generated by region S

$$= \pi \int_{0.077542}^{1} \left(\frac{1}{2}\ln y \right)^2 dy + \pi \int_{0}^{0.077542} \left(-1 - \sqrt{y} \right)^2 dy - \pi \int_{0}^{1} \left(-1 + \sqrt{y} \right)^2 dy$$

$$= 0.5593 \quad \text{(correct to 4 dec. place)}$$

(ci) When $y = 0$,
$$0 = \sin\frac{t}{2} + \cos\frac{t}{2}$$

$$\sin\frac{t}{2} = -\cos\frac{t}{2} \longrightarrow \tan\left(\tfrac{1}{2}t\right)$$

$$\tan\frac{t}{2} = -1$$

since factor of.
2 along x axis

$$t = -\frac{\pi}{2} \quad \text{since } -\frac{3\pi}{2} \le t \le -\frac{\pi}{2}$$

$$x = 2\left(\sin\left(-\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{4}\right) \right) = 2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) = -2\sqrt{2}$$

(ii) Area $= -\displaystyle\int_{-2\sqrt{2}}^{-2} y_C \, dx - \frac{1}{2}\left(\frac{1}{2}\right)(1)$

$$= -\int_{-\frac{\pi}{2}}^{-\pi} \left[\sin\frac{t}{2} + \cos\frac{t}{2} \right]\left(2\left(\sin\frac{t}{2} + \cos\frac{t}{2} \right) \right) dt - \frac{1}{4}$$

$$= \int_{-\pi}^{-\frac{\pi}{2}} \left(\sin\left(\frac{t}{2}\right) + \cos\left(\frac{t}{2}\right) \right)^2 dt - \frac{1}{4}$$

$$= \int_{-\pi}^{-\frac{\pi}{2}} \left[\sin^2\left(\frac{t}{2}\right) + \cos^2\left(\frac{t}{2}\right) + 2\sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right) \right] dt - \frac{1}{4}$$

Commented [KML24]: Things to Note
1) Most students knew that they had to use the factor formula Involving difference in 2 cosines to integrate $\sin\frac{x}{2}\cdot\sin\frac{3x}{2}$ but they made mistakes in the expression involving the 2 cosines especially with the angles or signs.
2) Most students didn't know how to integrate the terms in cot 2x even after they obtained $\cot^4 2x \left(\csc^2 2x \right)$.

Commented [KML25]: Presentation
Many didn't show clear working on how they obtained $t = -\frac{\pi}{2}$ from $y = 0$.

They straightaway wrote down $t = -\frac{\pi}{2}$ from $y = 0$.

Commented [KML26]: Presentation
As this is a "show" question, students need to show clearly the substitution to $-\frac{1}{\sqrt{2}}$ from $\sin\left(-\frac{\pi}{4}\right)$ and $\cos\left(-\frac{\pi}{4}\right)$.

Commented [KML27]: Presentation
Many didn't show the integral in cartesian form and straightaway changed it to the parametric form.
Misconception
Area must be positive.
1) Many missed out the minus sign in front of the integral involving y_C as $y_C < 0$.
2) Some put C in the integral instead of y_C.
3) Instead of finding the area of triangle straightaway, many used $\displaystyle\int_{-5/2}^{-2} (-2x - 5)\,dx$, missing out the minus sign in front of the integral.
4) Some didn't change the limits of the integral when they changed the variables from x to t.

$$= \int_{-\pi}^{\frac{-\pi}{2}} \left(1+\sin t\right) \, dt - \frac{1}{4}$$

$$= \left[t - \cos t \right]_{-\pi}^{\frac{-\pi}{2}} - \frac{1}{4}$$

$$= -\frac{\pi}{2} - \left(-\pi - \cos\left(-\pi\right)\right) - \frac{1}{4}$$

$$= \frac{\pi}{2} - \frac{5}{4}$$

Section B: Statistics [60 marks]

5 Elaine always receives two \$2 notes, two \$5 notes and one \$20 note from her parents as her monthly allowance. She decides to select two of these notes to contribute to Community Chest for the month of August. The total value of these notes is denoted by \$$C$.

(i) Determine the probability distribution of C. [3]

Commented [TCK28]: Quite a number of candidates did not read the question with care. First, this is selection of 2 notes without replacement; second, there is only one \$20 note!

(ii) Find the expected amount of money that she will donate in August and the variance of C. [2]

In September, Elaine decides to contribute to Community Chest again.

(iii) Find the probability that her total amount of donations for August and September is at least \$12. [2]

Solution

(i) Case 1: $P(\$2, \$2) = P(C = 4) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

Case 2: $P(\$2, \$5)$ or $P(\$5, \$2) = P(C = 7) = 2! \times \left(\frac{2}{5} \times \frac{2}{4} \right) = \frac{2}{5}$

Commented [TCK29]: Failure to take into consideration the factor '2!' - a \$2 note followed by a \$5 note or vice-versa.
<u>Recommendation</u> for those who tend to forget about the number of permutations – use combination

e.g. in Case 2, probability $= \dfrac{^2C_1 \times ^2C_1}{^5C_2} = \dfrac{2}{5}$

in Case 4, probability $= \dfrac{^1C_1 \times ^2C_1}{^5C_2} = \dfrac{1}{5}$

Case 3: $P(\$5, \$5) = P(C = 10) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

Case 4: $P(\$20, \$2)$ or $P(\$2, \$20) = P(C = 22) = 2! \times \left(\frac{1}{5} \times \frac{2}{4} \right) = \frac{1}{5}$

Case 5: $P(\$20, \$5)$ or $P(\$5, \$20) = P(C = 25) = 2! \times \left(\frac{1}{5} \times \frac{2}{4} \right) = \frac{1}{5}$

C	4	7	10	22	25
$P(C = r)$	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$

(ii) $E(C) = 4\left(\frac{1}{10}\right) + 7\left(\frac{2}{5}\right) + 10\left(\frac{1}{10}\right) + 22\left(\frac{1}{5}\right) + 25\left(\frac{1}{5}\right) = 13.6$

Expected amount of money = \$13.60
(Answer to 2 dp)

Commented [TCK30]: Reminder: Always leave answer in 2 d.p. if variable is money.

Variance of $C = E\left(C^2\right) - \left[E\left(C\right) \right]^2$ (with substitutions)

[Turn Over

$$E\left(C^2\right) = 4^2\left(\frac{1}{10}\right) + 7^2\left(\frac{2}{5}\right) + 10^2\left(\frac{1}{10}\right) + 22^2\left(\frac{1}{5}\right) + 25^2\left(\frac{1}{5}\right)$$
$$= 1.6 + 19.6 + 10 + 96.8 + 125$$
$$= 253$$

$\text{Var}(C) = 253 - (13.6)^2 = 68.04$

(iii) Complementary Method

Identify cases that will be less than $12 in donations :

1. $C_1 + C_2 = 2(\$4) = \$8$

2. $C_1 + C_2 = (\$4) + (\$7) = \$11$

$P(C_1 + C_2 \geq 12) = 1 - P(C_1 + C_2 < 12)$

$$= 1 - \left(\left(\frac{1}{10}\right)^2 + 2\left(\frac{2}{5}\right)\left(\frac{1}{10}\right)\right) = \frac{91}{100}$$

> **Commented [TCK31]:** Candidates should bear this useful method in mind. Only a few candidates who did the direct approach of finding total donations at least $12 succeed in getting the correct answer because of the sheer number of possible cases.
> Misconception: $C_1 + C_2$ is a discrete r.v., so it does make sense to attach a normal distribution to $C_1 + C_2$.
> Many candidates use this wrong approach to find the probability.

6 A group of 300 students are asked whether they own any earbuds, laptops or games machines.

90 students own a pair of earbuds
177 students own a laptop
100 students own a games machine

> **Commented [TCK32]:** Reading & Interpretation of question:
> 300 is the total number of students, not 90+177+100 = 367 as some candidates thought.

Events A, L and G are defined as follows:

A: a randomly chosen student owns a pair of earbuds.
L: a randomly chosen student owns a laptop.
G: a randomly chosen student owns a game machine.

It is given that events A and G are independent events. It is also given that 35 students own a laptop and a games machine and that 20 students own all the three gadgets.

(i) Find the probability that a student selected at random owns either a games machine or a pair of earbuds but not both. [2]

(ii) A student selected at random owns a games machine. Find the probability that the student owns exactly two of the gadgets. [2]

(iii) Find the greatest and least value of $P(A \cap L \cap G')$. [3]

> **Commented [TCK33]:** Reading & Interpretation of question:
> $P(A \cap G) = P(A) \times P(G) = \frac{90}{300} \times \frac{100}{300} = \frac{1}{10}$
> $\Rightarrow n(A \cap G) = \frac{1}{10} \times 300 = 30$
> $\Rightarrow n(A \cap G \cap L') = 30 - n(A \cap G \cap L)$
> $\qquad = 30 - 20$
> $\qquad = 10$
> Many candidates take $A \cap G$ to mean a student owns a pair of earbuds and games machine only, which is wrong.

Solution

$P(L) = \frac{177}{300} = \frac{59}{100}$

$P(G) = \frac{100}{300} = \frac{1}{3}$

$P(A) = \frac{90}{300} = \frac{3}{10}$

$P(G \cap L) = \frac{35}{300} = \frac{7}{60}$

$P(A \cap G \cap L) = \frac{20}{300} = \frac{1}{15}$

> **Commented [TCK34]:** Reading & Interpretation of question:
> $n(L \cap G) = 35$
> $n(L \cap G \cap A) = 20$
> $\Rightarrow n(L \cap G \cap A') = 35 - 20 = 15$
> Many candidates take $L \cap G$ to mean a student owns a laptop and games machine only, which is wrong.

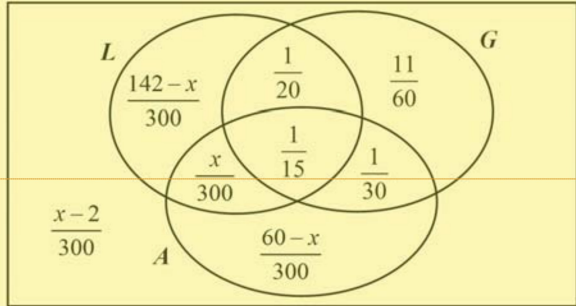> **Commented [TCK35]:** Reading & Interpretation of question:
> $n(L \cap G \cap A) = 20$

(i) Required probability $= \mathrm{P}(G \cup A) - \mathrm{P}(G \cap A) = \dfrac{100}{300} + \dfrac{90}{300} - 2\left(\dfrac{100}{300}\right)\left(\dfrac{90}{300}\right)$	
$= \dfrac{13}{30}$	
(ii) $\mathrm{P}(\text{owns two gadgets} \mid \text{owns a game machine}) = \dfrac{15 + 10}{100}$ or $\dfrac{\dfrac{1}{20} + \dfrac{1}{30}}{\dfrac{1}{3}}$	
$= \dfrac{1}{4}$	
(iii) Let $\mathrm{P}(L \cap A \cap G') = \dfrac{x}{300}$	

From the above, $2 \le x \le 60$ for all probabilities to be at least zero.

$\dfrac{2}{300} \le \mathrm{P}(A \cap L \cap G') \le \dfrac{60}{300}$

$\dfrac{1}{150} \le \mathrm{P}(A \cap L \cap G') \le \dfrac{1}{5}$

Commented [TCK36]: <u>Misconception</u>:
$G \cup A$ means a student selected randomly owns either a games machine or a pair of ear buds or both.

BUT question says 'a student selected at random owns either a games machine or a pair of earbuds **but not both**'. (Note italics)

Commented [TCK37]: <u>Reading & Interpretation of question</u>:
Question implies a condition probability in (ii).

$Prob. = \dfrac{\mathrm{P}(A \cap G \cap L') + \mathrm{P}(L \cap G \cap A')}{\mathrm{P}(G)}$

$= \dfrac{\dfrac{10}{300} + \dfrac{15}{300}}{\dfrac{1}{3}}$

$= \dfrac{\dfrac{1}{30} + \dfrac{1}{20}}{\dfrac{1}{3}}$

$= \dfrac{1}{4}$

Commented [TCK38]: <u>Notation</u>:
No proper use of notation is an issue.
e.g. the right way to notate a student who owns a laptop and a pair of earbuds nut no games machine is
$L \cap A \cap G'$.

e.g. $L \cap A' \cap G'$ is the event a student owns a laptop but no earbuds and games machine
NOT L only

e.g. $L' \cap A \cap G'$ is the event a student does not own all 3 gadgets.

Commented [TCK39]: <u>Reading & Interpretation of question</u>:
The main problem is that candidates did not consider that there may be students who do not own all the 3 gadgets. As a result, their analysis is wrong.

[**Turn Over**

7	The masses, in grams, of mangoes have the distribution $N(200, 30^2)$ and the masses, in grams, of oranges have the distribution $N(\mu, \sigma^2)$.	
	(i) Let A be the average mass, in grams, of a mango and two oranges. Given that $P(A < 162) = P(A > 206) = 0.06$, find μ. Hence form an equation involving σ and solve for σ.	[5]
	(ii) Find probability that the total mass of 3 mangoes differs from 3 times the mass of an orange by at most 136 grams.	[3]
	(iii) Mangoes are sold at \$26 per kilogram. Find the probability that the total selling price of 6 mangoes exceeds \$29.50.	[2]
	Solution	
	(i) Let X be the mass, in grams, of a randomly chosen mango. Let Y be the mass, in grams, of a randomly chosen orange. $X \sim N(200, 30^2)$ and $Y \sim N(\mu, \sigma^2)$	
	Let $A = \dfrac{X + Y_1 + Y_2}{3}$	
	$A \sim N\left(\dfrac{200 + 2\mu}{3}, \dfrac{2(\sigma^2) + 30^2}{9} \right)$	
	$P(A < 162) = P(A > 206) = 0.06$	
	$\Rightarrow \dfrac{200 + 2\mu}{3} = \dfrac{162 + 206}{2}$	
	$\Rightarrow \dfrac{200 + 2\mu}{3} = 184$	
	$\Rightarrow \mu = 176$	
	$P(A > 206) = 0.06$	
	$P\left(Z > \dfrac{206 - 184}{\sqrt{\dfrac{2\sigma^2 + 900}{9}}} \right) = 0.06 \quad \text{where } Z \sim N(0,1)$	
	$\dfrac{66}{\sqrt{2\sigma^2 + 900}} = 1.55477$ $2\sigma^2 + 900 = 1801.994907$ $\sigma = 21.237 \text{ (5 s.f.)} = 21.2 \text{ (3 s.f.)}$	
	(ii) Let $S = X_1 + X_2 + X_3 - 3Y$	
	$S \sim N(3 \times 200 - 3 \times 176, \ 3 \times 30^2 + 3^2 \times 21.237^2)$	
	$S \sim N(72, \ 6758.977)$	
	$P(|S| \le 136) = P(-136 < S < 136)$	
	$= 0.776 \text{ (3 s.f.)}$	

Commented [KML40]: Question Reading
Some took A to be the total mass instead of the average mass.

Commented [KML41]: Presentation
1) A large number of students didn't show this expression in X and Y.
2) Some didn't put in the distribution for A after finding E(A) and var(A).

Commented [KML42]: Mistake
A number of students divide by 3 instead of 3^2 in finding var(A).

Commented [KML43]: Misconception
Many put $\mu = \dfrac{162 + 206}{2}$ instead of E(A).

Commented [KML44]: Presentation and Misconception
Many didn't show this standardization involving Z and straightaway wrote the equation below involving invnorm.

In the standardization, many made the following mistakes:
1) $P\left(Z > \dfrac{206 - \mu}{\sigma} \right)$
2) $P\left(Z > \dfrac{206 - E(A)}{Var(A)} \right)$
instead of $P\left(Z > \dfrac{206 - E(A)}{\text{Std Dev}(A)} \right)$.

Commented [KML45]: Misconception
1) Many didn't put in the modulus sign.
2) A common mistake was
$P(|S| \le 136) = P(S > -136) + P(S < 136)$

	(iii) Let $T = 0.026\left(X_1 + X_2 + X_3 + X_4 + X_5 + X_6\right)$	
	$T \sim N\left(0.026 \times 6 \times 200,\ 0.026^2 \times 6 \times 30^2\right)$	
	$\quad T \sim N(31.2,\ 3.6504)$	
	$\quad P(T > 29.5) = 0.813\ (3\ \text{s.f.})$	

8	The boss of a hair salon chain claims that the waiting time, on average, is 15 minutes. Jeremy believes that the waiting time quoted by the hair salon chain is understated. He carries out a survey to investigate this by asking the waiting time for 110 random customers. The waiting time are summarised by	
	$$\sum t = 1900,\ \text{and}\ \sum(t-15)^2 = 25400.$$	
	(i) Find the unbiased estimate of the population mean. Leave your answers correct to 3 decimal places. Show that the unbiased estimate of the population variance is given by 227.815.	[2]
	(ii) Carry out a test on Jeremy's belief at the 5% significance level, stating a necessary assumption for the test.	[5]
	(iii) Kalai did another test to determine whether the mean waiting time differs from 15 minutes using another random sample. The waiting times of n randomly chosen customers, where n is large, were recorded, and their mean and standard deviation are 15.6 minutes and 2.5 minutes respectively. Given that the null hypothesis is rejected at the 5% level of significance, find the least value of n.	[4]
	Solution	
	(i) Unbiased estimate of population mean $= \dfrac{\sum t}{110} = \dfrac{1900}{110} = 17.273$	
	$\sum(t-15) = 1900 - 15(110) = 250,$	
	Unbiased estimate of population variance $= \dfrac{1}{109}\left[\sum(t-15)^2 - \dfrac{\left[\sum(t-15)\right]^2}{110}\right]$	
	$\dfrac{1}{109}\left[25400 - \dfrac{250^2}{110}\right] = 227.815$	
	(ii) Assume that the waiting time of a customer who visit the hair salon is independent of the other customers.	
	Let μ be the average waiting time of a customer.	
	Let T be the random variable "the waiting time of a randomly chosen customer".	
	To Test H_0: $\mu = 15$	
	Against H_1: $\mu > 15$	
	Left tailed z-test at 5% level of significance	
	Under H_0, since the sample size, 110, is large, by Central Limit Theorem,	
	$\overline{T} \sim N\left(15,\ \dfrac{227.815}{110}\right)$ approximately	

[**Turn Over**

Commented [KML46]: <u>Misconception</u>
1) Many didn't change to grams and worked with 26 instead of 0.026.
2) For those who made the change, a common mistake was they divide by 1000 instead of 1000^2 in finding var (T), i.e.
they put var(T) $= \dfrac{26^2}{1000} \times 6 \times 30^2$.

Commented [SH47]: <u>Recommendation</u>
Students are still unfamiliar with this topic. This is a show question so must working clearly for the calculation of the unbiased estimate of population variance.

Commented [SH48]: <u>Question Reading</u>
In 3 dp not in 3 sf!

Commented [SH49]: <u>Presentation</u>
Majority of students wrongly stated the assumption as "by Central Limit Theorem, since the sample size is 110, which is more than 30 and is sufficiently large, $\overline{T}$ will be normally distributed". This is an approximation, not an assumption.

So, students should pay attention and comment on the random variable T (which is the waiting time of a randomly chosen customer).

Commented [SH50]: <u>Presentation</u>
Careless use of Notations. Please pay attention to the use of the symbols as they would mean differently.
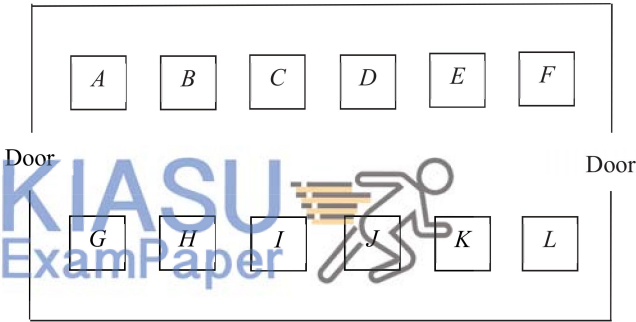
Commented [KSM51]: <u>Misconception</u>
Follow the logic of 'under Ho', the mean of $\overline{T}$ will be the pop mean value of 15!! Still many wrongly put it as the sample mean value.

Commented [SH52]: Students use $\overline{t}$ instead of $\overline{T}$. The test is for the population mean not for the sample mean.

	From the G.C, p value = 0.0571 (Accept value of higher accuracy)	
	Since p-value = 0.0571 > 0.05, we do not reject H_0 and conclude that there is insufficient evidence that the mean waiting time is more than 15 minutes at 5% significance level.	
	(iii) Let X denote the waiting time of a randomly chosen customer.	
	Unbiased estimate of population variance $= \dfrac{n}{n-1}(2.5)^2 = \dfrac{6.25n}{n-1}$	
	To Test H_0: $\mu = 15$ Against H_1: $\mu \neq 15$	
	2-tailed z-test at 5% level of significance (CLT	
	Under H_0, $\bar{X} \sim N\left(15, \dfrac{6.25n}{n-1}\right)$ approx.	
	Since H_0 is rejected,	
	$\dfrac{15.6-15}{\left(\sqrt{\dfrac{6.25n}{n-1}}\right)} < -1.96$ or $\dfrac{15.6-15}{\left(\sqrt{\dfrac{6.25n}{n-1}}\right)} > 1.96$	
	$\sqrt{n-1} < -8.1665$ (no solution) or $\sqrt{n-1} > 8.1665$	
	$n > 67.692$	
	Least n is 68.	
9	A rectangular barrack with a door at each end, has 12 beds marked A, B, C,..., L as shown in the diagram below. Johnson and 11 other soldiers take 1 bed each.	
	(i) Find the number of different sleeping arrangements possible if none of Johnson and 2 other particular soldiers are adjacent to each other, and all three of them are on the same side of the barrack.	[2]
	One afternoon, 9 particular soldiers went out to have lunch. There were 3 identical round tables at the dining venue.	
	(ii) How many ways can 9 soldiers be seated if there must be at least two soldiers at each table?	[4]

Commented [SH53]: Presentation Students need to follow structure on the conclusion. Many of the students provided the conclusion on the basis of H_0. Which is incorrect and that is the reason also why many said there is sufficient evidence.

Commented [SH54]: Question Reading Sample deviation is given implies sample variance is given. Almost all the students did not calculate the unbiased estimate for population variance.

Commented [SH55]: Presentation Must show rejection at the left tail.

Johnson was playing with a deck of 10 cards. When the cards are arranged in a certain manner, it forms the word SUCCESSFUL.

(iii)	Find the number of ways he can arrange the 10 cards such that not all the letter "S" are together.	[2]
(iv)	He wishes to replace his existing handphone 4-letter codeword using letters from the word SUCCESSFUL, how many different options would he have.	[4]

Solution

(i) Case 1:

The 3 individuals are in the top row.
Since they must be separated, we will slot them among the 3 remaining people in that row. There are in total 4 possible slots.

No. of ways $= \underbrace{^4C_3}_{\text{choose the 3 slots}} \times \underbrace{3!}_{\substack{\text{arrange among the} \\ \text{3 individuals}}} \times \underbrace{9!}_{\substack{\text{arrange the other} \\ \text{9 people}}}$

Case 2: The 3 individuals are in the bottom row.
Same number of ways as Case 1.

Total number of ways $= 2 \times \left[^4C_3 \times 3! \times 9! \right] = 17418240$.

> **Commented [LMH56]:** Things to note
> Instead of writing 9! (to arrange all the other people), some students wrote 3! x 6! instead. These students have probably forgotten to <u>choose</u> who are the 3 soldiers who will be on the same side as Johnson and friends. They would be correct if they considered that, as
> $^9C_3 \times 3! \times 6! = 9!$.

(ii) Case 1: 2, 2, 5 people

No. of ways $= \left(\dfrac{^9C_2 \times ^7C_2}{2!} \times ^5C_5 \right) \times 1! \times 1! \times 4! = 9072$

Case 2: 2, 3, 4 people
No. of ways $= ^9C_2 \times ^7C_3 \times 1! \times 2! \times 3! = 15120$

Case 3: 3, 3, 3 people
No. of ways $= \dfrac{^9C_3 \times ^6C_3 \times ^3C_3}{3!} \times (2!)^3 = 2240$

Total no. of ways $= 9072 + 15120 + 2240 = 26432$

> **Commented [LMH57]:** Interpretation
> Some students forgot the need to arrange them around the tables. Also, some students forgot to divide by 2! or 3! to account for the identical group sizes. Note that the tables are identical.

(iii)

Number of ways
= Number of ways without restrictions − number of ways that the SSS group is together

$= \underbrace{\dfrac{10!}{2! \, 2! \, 3!}}_{\substack{10 \text{ letters} \\ 2 \text{ C's 2U's 3 S's}}} - \underbrace{\dfrac{8!}{2! \, 2!}}_{\substack{8 \text{ groups} \\ 2 \text{ C's 2U's}}}$

$= 141120$

> **Commented [LMH58]:** Question Reading
> This is similar to the P&C question in MYCT. Yet significant number of students still couldn't understand "not all the 3 S are together" and proceeded to slot them.
>
> If question wants you to separate all the 'S', this might be phrased as: "no two S are together". In this case, yes, go ahead and use slotting method.

(iv)

Case I: 3 identical letters

Select 1 letter from U, C, E, F, L

No of 4-letter code-words with the chosen letter and 3 "S"s

$$= {}^5C_1 \times \frac{4!}{3!} = 20$$

Case II: 1 pair of identical letters

Select 1 pair from UU, CC, SS + select 2 distinct letters from remaining letters

No of 4-letter code-words with the chosen pair of same letters and the chosen 2 letters

$$= {}^3C_1 \times {}^5C_2 \times \frac{4!}{2!} = 360$$

Case III: 2 pairs of same letters

Select 2 pairs from UU, CC, SS and select 2 more distinct letters from remaining

No. of such code-words $= {}^3C_2 \times \frac{4!}{2!2!} = 18$

Case IV: All distinct letters

Select 4 letters from S, U, C, E, F, L

No. of such code-words $= {}^6C_4 \times 4! = 360$

Total no. of 4-letter code-words $= 20 + 360 + 18 + 360 = 758$

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10 A ferry company operates a 9am ferry, with a passenger capacity of 108, from Island X to Island Y daily. To maximise profits, 113 tickets are sold online each day because it was found that on average, $p\%$ of customers who have purchased a ticket do not turn up.

(i) It is known that there is a probability of 0.012 that at most 1 customer will not turn up for the ferry. Write down an equation in terms of p, and hence find p correct to 3 decimal places. [3]

It is now given that $p = 6$.

(ii) Find the probability that there are no empty seats if every customer who turns up get a seat on the 9am ferry. [3]

(iii) Find the probability that, in a randomly chosen week, every customer who turns up gets a seat on the 9am ferry on at least 5 days of the week. [2]

(iv) Find the maximum number of tickets that could be sold so that the probability of a customer not getting a seat on the ferry is no more than 0.01. [3]

(v) A period of forty days are randomly selected. Find the probability that the mean daily number of customers who purchased a ticket and managed to turn up during this period is at most 106. [2]

Solution

(i)

Let X be the random variable denoting the number of customers who did not turn up for the 9 am ferry out of 113 customers on a randomly chosen day.

$$X \sim B\left(113, \frac{p}{100}\right)$$

Commented [LMH59]: Misconception
Note that when you use ${}^nC_r$, the objects you are choosing from must be **distinct**. It is clear that several students are not aware of this fact.

Commented [NCY60]: Presentation
Random variable should always be defined clearly and distribution must always be stated.

Commented [NCY61]: Question reading
P% ➔ probability = $\dfrac{p}{100}$

Commented [LT62]: Question Reading
A number of students did not take note of the question requirement.

Commented [LT63]: Question Reading
Many did not read this part of the statement carefully and so failed to see that it is a conditional probability question.

Commented [LT64]: Misconception
Some students defined the random variable that is out of 108 customers.

Commented [LT65]: Misconception
The parameters for Binomial Distribution are number of trials and **probability of success**. So it is incorrect to write it as $X \sim B(113, p)$

It is also incorrect to write $p = \dfrac{p}{100}$ in the working as it would mean $1 = \dfrac{1}{100}$ for non-zero p.

$\text{P}(X \le 1) = 0.012$

$\text{P}(X = 0) + \text{P}(X = 1) = 0.012$

$\left(1 - \dfrac{p}{100}\right)^{113} + 113\left(\dfrac{p}{100}\right)\left(1 - \dfrac{p}{100}\right)^{112} = 0.012$

Using GC, $p = 5.554$ (3 decimal place)

(ii) Let Y be the random variable that denotes the number of customers who purchased a ticket and turn up for the 9 am ferry out of 113 customers.
$Y \sim \text{B}(113, 0.94)$

$\text{P}(Y \ge 108 \mid Y \le 108)$

$= \dfrac{\text{P}(Y = 108)}{\text{P}(Y \le 108)}$

$= \dfrac{0.136717}{0.814523}$

$= 0.168$

(iii) Let W be the random variable that denotes the number of days where every customer who turns up get a seat on the 9 am ferry out of 7 days.

$W \sim \text{B}(7, 0.814523)$

$\text{P}(W \ge 5) = 1 - \text{P}(W \le 4) = 0.876$

(iv) Let T denote the number of customers who purchased a ticket and turn up for the 9am ferry out of n passengers.

$T \sim \text{B}(n, 0.94)$
$\text{P}(T > 108) \le 0.01$
$\text{P}(T \le 108) \ge 0.99$

Using GC,
When $n = 109$, $\text{P}(T \le 108) = 0.99882 > 0.99$
When $n = 110$, $\text{P}(T \le 108) = 0.99112 > 0.99$
When $n = 111$, $\text{P}(T \le 108) = 0.96571 < 0.99$

maximum $n = 110$

(v) Let Y be the random variable that denotes the number of customers who purchased a ticket and turn up for the 9 am ferry out of 113 customers.

$Y \sim \text{B}(113, 0.94)$
Since $n = 40$ is large, by Central Limit Theorem,
$\overline{Y} \sim \text{N}(106.22, 0.15933)$ approximately.

$\text{P}(\overline{Y} \le 106) = 0.291$

End of Paper

[**Turn Over**

Commented [LT66]: Misconception
It is incorrect to apply the formula inside MF 26 directly. It is important to note that the probability has to be of the form $\text{P}(X = k)$ before applying the formula.

Commented [LT67]: Question Reading
Many cannot see that "every customer who turns up gets a seat" means $Y \le 108$.

Commented [LT68]: Accuracy of calculation
It is important to leave all intermediate answers correct to 5 significant figures.

Commented [LT69]: Notation
Do not reuse the same letter for different definitions.

Recommendation
Do not use the following letters:
B – For binomial distribution
P – For probability
O – Can be confused with origin or zero
N – Normal Distribution
Z – For Standard Normal
E – For Expectations

It is important to define the random variable properly for the subsequent workings to make sense.

Commented [LT70]: Question Reading
Many wrote down the incorrect probability statement.
For example
  1. $\text{P}(T > 108) < 0.01$
  2. $1 - \text{P}(T < 108) \le 0.01$

Commented [LT71]: Misconception
It is incorrect to write
$Y \sim \text{N}(106.22, 0.15933)$.

It is also important to note that the distribution obtained is an approximate Normal Distribution under Central Limit Theorem.