

## 2017 MATHEMATICS HIGHER 2 SPECIMEN PAPER (SYLLABUS 9758) Paper 2

## Section A: Pure Mathematics [40 marks]

- 1 (i) The function
- $f$
- is defined as follows:

$$f: x \mapsto 3 \cos x - 2 \sin x, \quad x \in \mathbb{R}, -\pi \leq x < \pi.$$

Write  $f(x)$  as  $R \cos(x + \alpha)$ , where  $R$  and  $\alpha$  are constants to be found. Hence, or otherwise, find the range of  $f$  and sketch the curve. [4]

- (ii) The function
- $g$
- is defined as follows:

$$g: x \mapsto 3 \cos x - 2 \sin x, \quad x \in \mathbb{R}, -\alpha \leq x \leq b.$$

Given that the function  $g^{-1}$  exists, write down the largest value of  $b$ . Find  $g^{-1}(x)$ . [3]

- 2 The first four terms of a sequence of numbers are 3, 1, 1 and 3.
- $S_n$
- is the sum of the first
- $n$
- terms of this sequence.

- (i) Explain why
- $S_n$
- cannot be a quadratic polynomial in
- $n$
- . [2]

It is given that  $S_n$  is a cubic polynomial.

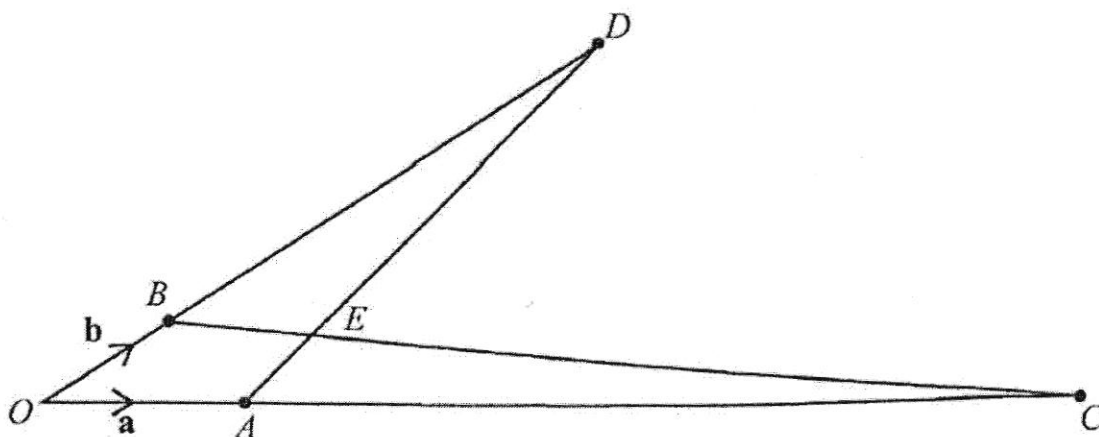
- (ii) Find
- $S_n$
- in terms of
- $n$
- . [4]

- (iii) Find an expression in terms of
- $n$
- for the
- $n$
- th term of the sequence. [3]

- 3 (a) The angle between the vectors
- $3\mathbf{i} - 2\mathbf{j}$
- and
- $6\mathbf{i} + d\mathbf{j} - \sqrt{7}\mathbf{k}$
- is
- $\cos^{-1}\left(\frac{6}{13}\right)$
- .

Show that  $2d^2 - 117d + 333 = 0$ . [3]

- (b)



With reference to the origin  $O$ , the points  $A$ ,  $B$ ,  $C$  and  $D$  are such that  $\overline{OA} = \mathbf{a}$ ,  $\overline{OB} = \mathbf{b}$ ,  $\overline{AC} = 5\mathbf{a}$  and  $\overline{BD} = 3\mathbf{b}$ . The lines  $AD$  and  $BC$  cross at  $E$  (see diagram).

- (i) Find  $\overline{OE}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ . [6]  
 (ii) The point  $F$  divides the line  $CD$  in the ratio 5 : 3. Show that  $O$ ,  $E$  and  $F$  are collinear, and find  $OE:OF$ . [4]

- 4 (i) Given that  $y = \tan(e^{2x} - 1)$ , show that  $\frac{dy}{dx} = ke^{2x}(1 + y^2)$ , where  $k$  is to be found. Hence find the values of  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$  and  $\frac{d^3y}{dx^3}$  when  $x = 0$ . [6]
- (ii) Write down the first three non-zero terms in the Maclaurin series for  $\tan(e^{2x} - 1)$ . [1]
- (iii) The first two non-zero terms in the Maclaurin series for  $\tan(e^{2x} - 1)$  are equal to the first two non-zero terms in the series expansion of  $e^{ax} \ln(1 + nx)$ . By using appropriate expansions from the List of Formulae (MF 26), find the constants  $a$  and  $n$ . Hence find the third non-zero term of the series expansion of  $e^{ax} \ln(1 + nx)$  for these values of  $a$  and  $n$ . [4]

**Section B: Probability and Statistics [60 marks]**

- 5 This question is about six couples. Each couple consists of a husband and wife. The 12 people visit a theatre, and sit in a row of 12 seats.
- (i) In how many different ways can the 12 people sit so that each husband and wife in a couple sit next to each other? [2]
- (ii) In how many different ways can the 12 people sit so that the 6 wives all sit next to each other, and none of the wives sits next to her own husband? [3]

The group decides to form a committee to arrange future outings. The committee will consist of 3 of the 12 people. At least 1 of the wives will be on the committee but no husband and wife couple will be included.

- (iii) In how many ways can the committee be formed? [3]
- 6 Giant pumpkins are often irregular in shape. In order to account for the different shapes of pumpkins, growers of giant pumpkins measure the size of a pumpkin by a combination of three measurements, called the 'over the top' length. Pumpkin growers keep records so that they can estimate the mass of giant pumpkins while they are still growing. The over the top lengths ( $d$  m) and the masses ( $m$  kg) of a random sample of 7 giant pumpkins are as follows.

$d$	2.31	2.9	4.05	5.5	6.7	7.92	9.17
$m$	11	14	47	104	170	282	449

- (i) Draw a scatter diagram of these data, and explain how you know from your diagram that the relationship between  $m$  and  $d$  should not be modelled by an equation of the form  $y = ax + b$ . [2]
- (ii) Which of the formulae  $m = ed^2 + f$  and  $m = gd^3 + h$ , where  $e, f, g$  and  $h$  are constants, is the better model for the relationship between  $m$  and  $d$ ? Explain fully how you decided, and find the constants for the better formula. [5]
- (iii) Use the formula you chose from part (ii) to estimate the mass of a giant pumpkin with  
 (a) over the top length 6 m,  
 (b) over the top length 12m.  
 Explain which of your two estimates is more reliable. [3]

7 'Bings' are sweets that are sold in packets of 6. Each packets is made up of randomly chosen coloured sweets. On average 10% of Bings are yellow.

(i) Explain why a binomial distribution is appropriate for modelling the number of yellow sweets in a packet. Find the probability that a randomly chosen packet of Bings contains no more than one yellow sweet. [3]

(ii) Kev buys 90 randomly chosen packets of Bings. Find the probability that at least 80 of these packets contain no more than one yellow sweet. [2]

On average the proportion of Bings that are red is  $p$ . It is known that the modal number of red sweets in a packet is 2.

(iii) Use this information to find exactly the range of values that  $p$  can take. [4]

8 A bag contains 3 blue counters, 1 red counter and  $y$  yellow counters. Darvina chooses 3 counters at random from the bag, without replacement. The random variable  $S$  is the sum of the number of blue counters chosen and **twice** the number of red counters chosen.

(i) Show that  $P(S = 3) = \frac{6(3y+1)}{(y+4)(y+3)(y+2)}$ . [2]

(ii) Given that  $P(S = 3) = \frac{7}{20}$ , calculate  $y$ . Hence find the probability distribution of  $S$ . [6]

9 A type of metal bolt is manufactured with a nominal radius of 0.8cm. In fact, the radii of the bolts, is measured in cm, have the distribution  $N(0.8, 0.01^2)$ .

(i) Find the percentage of bolts that have a radius between 0.79cm and 0.82cm. [1]

Metal washers are manufactured to fit on the bolts. The inside radii of the washers, measured in cm, have the distribution  $N(0.81, 0.012^2)$ .

(ii) Write down the distribution of the inside circumference of the washers, in cm, and find the circumference that is exceeded by 5% of the washers. [4]

A bolt and a washer are 'a good fit' if

- the inside radius of the washer is greater than the radius of the bolt and
- the inside radius of the washer is not more than 0.04 cm greater than the radius of the bolt.

(iii) A washer is chosen at random, and a bolt is chosen at random. Find the probability that the washer and the bolt is a good fit. [3]

The outside radii of the washers, measured in cm, have the distribution  $N(\mu, \sigma^2)$ . It is known that 15% of the washers have an outside radii greater than 1.25cm and 25% have an outside radius of less than 1.15cm.

(iv) Find the values of  $\mu$  and  $\sigma$ . [4]

- 10 The average time required for the manufacture of a certain type of electronic control panel is 17 hours. An alternative manufacturing process is trialled, and the time taken,  $t$  hours, for the manufacture of each of the 50 randomly chosen panels using the alternative process is recorded. The results are summarised as follows.

$$n = 50 \quad \sum t = 835.7 \quad \sum t^2 = 14067.17$$

The Production Manager wishes to test whether the average time taken for the manufacture of a control panel is different using the alternative process, by carrying out a hypothesis test.

- (i) Explain whether the Production Manager should carry out a 1-tail test or a 2-tail test. [1]
- (ii) Explain why the Production Manager is able to carry out a hypothesis test without knowing anything about the distribution of the times taken to manufacture the control panels. [2]
- (iii) Find unbiased estimates of the population mean and variance and carry out the test at the 10% level of significance for the Production Manager. [6]
- (iv) Suggest a reason why the Production Manager might be prepared to use an alternative process that takes a longer average time than the original process. [1]

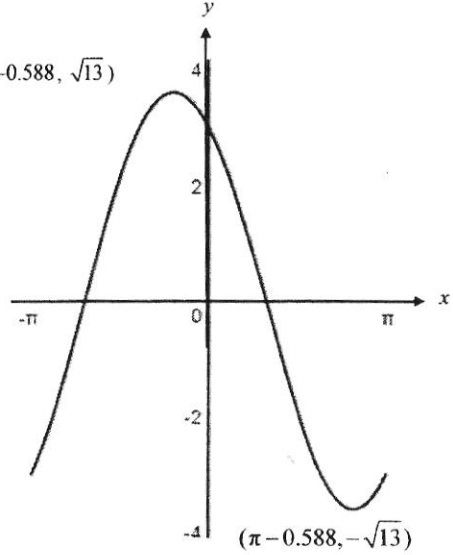
The Finance Manager wishes to test whether the average time taken for the manufacture of a control panel is **shorter** using the alternative process. The Finance Manager finds that the average time taken for the manufacture of each of 40 randomly chosen control panels, using the alternative process, is 16.7 hours. He carries out a hypothesis test at the 10% level of significance.

- (v) Explain, with justification, how the population variance of the times will affect the conclusion made by the Finance Manager. [3]

**END OF PAPER**

GCE A Level H2 Mathematics Specimen Paper 2 (Mark Scheme)

Qn	Answer	Guidance
1(i)	$f(x) = 3 \cos x - 2 \sin x$ M1: Recall R-formula: $a \cos x - b \sin x = \sqrt{a^2 + b^2} \cos\left(x + \tan^{-1} \frac{b}{a}\right)$ $R = \sqrt{13}, \alpha = \tan^{-1}\left(\frac{2}{3}\right) = 0.588$ M2: Compound angle for cosine $R \cos(x + \alpha) = R \cos x \cos \alpha - R \sin x \sin \alpha$ $= R \cos \alpha \cos x - R \sin \alpha \sin x$ $R \cos \alpha = 3, R \sin \alpha = 2$ $(R \cos \alpha)^2 + (R \sin \alpha)^2 = R^2$ $3^2 + 2^2 = R^2$ $R = \sqrt{13}$ $\frac{R \sin \alpha}{R \cos \alpha} = \frac{2}{3}$ $\tan \alpha = \frac{2}{3}$ $\alpha = \tan^{-1}\left(\frac{2}{3}\right) = 0.588$ $f(x) = \sqrt{13} \cos(x + 0.588)$ $R = \sqrt{13}, \alpha = 0.588$	M1: Attempt to use R formula A1 For both R and $\alpha$ . M1: working to solve for R A1 For both R and $\alpha$ .

	Sketch with max at $(-0.588, \sqrt{13})$ and min at $(\pi - 0.588, -\sqrt{13})$ 	B1 cos curve with max at $(-0.588, \sqrt{13})$ and min at $(\pi - 0.588, -\sqrt{13})$
	Range of f is $[-\sqrt{13}, \sqrt{13}]$	B1
1(ii)	Largest value of b is $\pi - \alpha$ . $y = \sqrt{13} \cos\left(x + \tan^{-1} \frac{2}{3}\right)$ $x = \cos^{-1} \frac{y}{\sqrt{13}} - \tan^{-1} \frac{2}{3}$ $g^{-1}(x) = \cos^{-1}\left(\frac{x}{\sqrt{13}}\right) - 0.588$	B1 M1: Attempt to make x the subject. A1

Qn	Answer	Guidance
2 (i)	$T_n$ is always one degree less than $S_n$ since $T_n = S_n - S_{n-1}$ . $S_n$ quadratic would imply that $T_n$ is linear but $T_n$ is not.	B1 B1
(ii)	$S_n = an^3 + bn^2 + cn + d$	M1 one set of values substituted. M1 3, 4, 5, 8

	$S_1 = a + b + c + d = 3$ $S_2 = 8a + 4b + 2c + d = 4$ $S_3 = a(3)^3 + b(2)^2 + c(2) + d = 5$ $S_4 = a(4)^3 + b(4)^2 + c(4) + d = 8$ <p>Using GC: <math>S_n = \frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n</math></p>	<p><b>A1</b> All equations are correct</p> <p><b>A1</b></p>
(iii)	$T_n = S_n - S_{n-1}$ $= \frac{1}{3}n^3 - 2n^2 + \frac{14}{3}n - \left[ (n-1)^3 - 2(n-1)^2 + \frac{14}{3}(n-1) \right]$ $= n^2 - 5n + 7$	<p><b>M1:</b> Attempt to find <math>S_n - S_{n-1}</math></p> <p><b>A1</b> Expression for <math>S_{n-1}</math></p> <p><b>A1</b></p>
3(a)	$\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ d \\ -\sqrt{7} \end{pmatrix} \cos\left(\cos^{-1}\frac{6}{13}\right)$ $18 - 2d = \sqrt{13}\sqrt{36 + d^2} + 7\left(\frac{6}{13}\right)$ $\sqrt{13}\left(3 - \frac{d}{3}\right) = \sqrt{43 + d^2}$ $13\left(3 - \frac{d}{3}\right)^2 = 43 + d^2$ $13\left(9 + \frac{d^2}{9} - 2d\right) = 43 + d^2$ $117 + \frac{13d^2}{9} - 26d = 43 + d^2$ $117 + \frac{13d^2}{9} - 26d = 43 + d^2$ $2d^2 - 117d + 333 = 0$	<p><b>M1:</b> Attempt to apply <math>\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}  \mathbf{b} \cos\theta</math></p> <p><b>A1:</b> Simplification and answer.</p>
(b) (i)	$\frac{\overline{BC}}{\overline{AD}} = \frac{-\mathbf{b} + 6\mathbf{a}}{4\mathbf{b} - \mathbf{a}}$ <p>At E, <math>\mathbf{a} + \lambda(4\mathbf{b} - \mathbf{a}) = \mathbf{b} + \mu(-\mathbf{b} + 6\mathbf{a})</math></p> $\lambda = \frac{5}{23}$ $\mu = \frac{3}{23}$ $\overline{OE} = \frac{18}{23}\mathbf{a} + \frac{20}{23}\mathbf{b}$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>

(ii)	$\overline{OF} = 6\mathbf{a} + \frac{5}{8}(-6\mathbf{a} + 4\mathbf{b})$ $\overline{OF} = \frac{9}{4}\mathbf{a} + \frac{5}{2}\mathbf{b}$ <p>O, E and F are collinear since <math>\overline{OF} = k\overline{OE}</math>.</p> $OE = OF = 8:23$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>A1</b></p>
4(i)	$y = \tan(e^{2x} - 1)$ $\frac{dy}{dx} = 2e^{2x} \sec^2(e^{2x} - 1)$ $= 2e^{2x} [1 + \tan^2(e^{2x} - 1)]$ $= 2e^{2x} [1 + y^2]$ $k = 2$ $\frac{dy}{dx} = 2e^{2x} [1 + y^2]$ $\frac{d^2y}{dx^2} = 4e^{2x} [1 + y^2] + 2y \frac{dy}{dx} 2e^{2x}$ $= 2 \frac{dy}{dx} + 4e^{2x} y \frac{dy}{dx}$ $= 2 \frac{dy}{dx} (1 + 2e^{2x} y)$ $\frac{d^3y}{dx^3} = 2 \frac{d^2y}{dx^2} (1 + 2e^{2x} y) + 2 \frac{dy}{dx} (4e^{2x} y + 2e^{2x} \frac{dy}{dx})$ <p>When <math>x=0</math>,</p> $y = 0$ $\frac{dy}{dx} = 2$ $\frac{d^2y}{dx^2} = 4$ $\frac{d^3y}{dx^3} = 8 + 16 = 24$	<p><b>M1:</b> Attempt to differentiate</p> <p><b>A1</b></p> <p><b>M1:</b> Attempt to differentiate</p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>
(ii)	$\tan(e^{2x} - 1) \approx 2x + \frac{4}{2}x^2 + \frac{24}{3!}x^3 = 2x + 2x^2 + 4x^3$	<p><b>B1</b></p>

(iii)	$e^{ax} \ln(1+nx)$ $\approx \left( 1 + ax + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} \right) \left( nx - \frac{(nx)^2}{2} + \frac{(nx)^3}{3} \right)$ $\approx \left( nx - \frac{(nx)^2}{2} + anx^2 \right)$ $= nx + \left( an - \frac{n^2}{2} \right) x^2$ <p>Comparing the above with <math>2x + 2x^2</math>: <math>a = 2, n = 2</math></p> <p>Coefficient = <math>\frac{8}{3}</math></p>	<p><b>M1:</b> Attempt to apply the series expansion</p> <p><b>A1 A1</b></p> <p><b>B1</b></p>
5(i)	$6 \times 2^6 = 46080$	<p><b>M1:</b> <math>6 \times 2^6</math></p> <p><b>A1</b></p>
(ii)	<p><u>Wives together at one end</u></p> <div style="display: flex; justify-content: space-around; border: 1px solid black; padding: 5px;"> <div style="border: 1px solid black; padding: 2px;">Ordering of 6 wives</div> <div style="border: 1px solid black; padding: 2px;">5 options for one slot</div> <div style="border: 1px solid black; padding: 2px;">Ordering of 5 left over husbands</div> </div> <p><math>6 \times 5 \times 5! \times 2 = 864000</math> -- (1) (‘2’ is for having the wives at two different ends)</p> <p><u>Wives not at end of the row</u></p> <div style="display: flex; justify-content: space-around; border: 1px solid black; padding: 5px;"> <div style="border: 1px solid black; padding: 2px;">----- 5 options for one slot</div> <div style="border: 1px solid black; padding: 2px;">Ordering of 6 wives. Suppose the order is <math>W_2 \dots W_4</math></div> <div style="border: 1px solid black; padding: 2px;"><math>H_2</math> -----</div> </div> <p><math>5 \times 6! \times 1 \times 4 \times 5</math> -- (2)</p> <p>4!: ordering of the left over of 4 husbands 5: you can fill up the two boxes of four slots in the following manner =&gt; 4 + 0, 3 + 1, 2 + 2, 1 + 3, 0 + 4</p> <div style="display: flex; justify-content: space-around; border: 1px solid black; padding: 5px;"> <div style="border: 1px solid black; padding: 2px;">----- 4 choices</div> <div style="border: 1px solid black; padding: 2px;">Ordering of 6 wives. Suppose the order is <math>W_2 \dots W_4</math></div> <div style="border: 1px solid black; padding: 2px;">Not <math>H_2</math> 4 choices</div> <div style="border: 1px solid black; padding: 2px;">-----</div> </div> <p><math>4 \times 6! \times 4 \times 4 \times 5</math> -- (3)</p>	<p><b>M1:</b> Wives together at one end</p> <p><b>M1:</b> Wives not at end of row.</p>

	<p>4!: ordering of the left over of 4 husbands 5: you can fill up the two boxes of four slots in the following manner =&gt; 4 + 0, 3 + 1, 2 + 2, 1 + 3, 0 + 4</p> <p>Total number of ways : Adding expression (1) + (2) + (3) = 2678400</p>	<b>A1</b>
(iii)	$\binom{12}{3} - 6 \times 10 - \binom{6}{3} = 140$ <p><math>\binom{12}{3}</math>: unrestricted ways to select 3 people from 12 <math>6 \times 10</math>: no. of ways to select the 3 people with a couple included <math>\binom{6}{3}</math>: No. of ways to select such that all 3 people are husbands.</p>	<p><b>M1:</b> One correct term</p> <p><b>A1:</b> Individual terms are correct</p> <p><b>A1:</b> 140</p>
6(i)	<p>The scatterplot shows that as <math>d</math> increases, <math>m</math> increases at an increasing rate.</p>	<p><b>B1:</b> Diagram is not a straight line</p> <p><b>B1:</b> <math>d</math> increases, <math>m</math> increases at an increasing rate.</p>
(ii)	<p>For the cubic model: <math>r = 0.9995</math> For the square model: 0.9889 Since <math> r </math> is closer to 1 in the cubic model compared to the square model, it has a strong linear relationship. <math>m = 0.572d^3 + 3.74</math></p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1, B1</b></p>
(iii)	<p>(a) 127.3 (b) 992.2 The first one, since the second one is outside the range of the given [2.31, 9.17].</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>
7(i)	<p>The probability of a sweet being yellow is constant (0.1). A sweet is either yellow or not yellow – two outcomes.</p>	<b>B1</b>

	<p>The event of a sweet being yellow is independent of the other sweet being yellow.</p> <p>Let <math>x</math> be the number of yellow sweets in a packet</p> <p><math>x \sim \text{Binomial}(6, 0.1)</math></p> <p><math>P(x=0) + P(x=1)</math></p> <p><math>= 0.885735</math> (using GC Binomial cdf)</p> <p><math>= 0.886</math></p> <p>[Ref: <math>\binom{6}{0} 0.1^0 (1-0.1)^6 - \binom{6}{1} 0.1^1 (1-0.1)^5</math>]</p>	<p>MI: Use of <math>x \sim \text{Binomial}(6, 0.1)</math></p> <p>AI</p>
(ii)	<p>Let <math>y</math> be the number of packets with no more than one yellow sweet.</p> <p><math>y \sim \text{Binomial}(90, 0.885735)</math></p> <p>Probability</p> <p><math>= 1 - P(y \leq 79)</math></p> <p><math>= 1 - 0.45448858</math></p> <p><math>= 0.5455114</math></p> <p><math>= 0.546</math> (3sf)</p>	<p>MI: <math>1 - P(y \leq 79)</math></p> <p>AI</p>
(iii)	<p>Let <math>w</math> be the number of sweets that are red.</p> <p><math>P(w=k)</math></p> <p><math>= \binom{6}{k} p^k (1-p)^{6-k}</math></p> <p>Since 2 is the mode,</p> <p><math>P(w=2) &gt; P(w=1)</math> and <math>P(w=2) &gt; P(w=3)</math>.</p> <p><math>\binom{6}{2} p^2 (1-p)^4 &gt; \binom{6}{1} p^1 (1-p)^5</math></p> <p><math>\Rightarrow \binom{6}{1} p^1 (1-p)^5 - \binom{6}{2} p^2 (1-p)^4 &lt; 0</math></p> <p><math>\Rightarrow p^1 (1-p)^4 \left[ \binom{6}{1} (1-p) - \binom{6}{2} p \right] &lt; 0</math></p> <p><math>\Rightarrow p^1 (1-p)^4 (6-6p-3p) &lt; 0</math></p> <p><math>\Rightarrow p &gt; \frac{2}{7}</math> (since <math>p^1 (1-p)^4 &gt; 0</math>)</p>	<p>MI</p> <p><math>P(w=2) &gt; P(w=1)</math></p> <p>AI</p>

	<p><math>\binom{6}{2} p^2 (1-p)^4 &gt; \binom{6}{3} p^3 (1-p)^3</math></p> <p><math>\Rightarrow \binom{6}{3} p^3 (1-p)^3 - \binom{6}{2} p^2 (1-p)^4 &lt; 0</math></p> <p><math>\Rightarrow p^2 (1-p)^3 \left[ \binom{6}{3} p - \binom{6}{2} (1-p) \right] &lt; 0</math></p> <p><math>\Rightarrow p^2 (1-p)^3 [20p - 15 + 15p] &lt; 0</math></p> <p><math>\Rightarrow p &lt; \frac{3}{7}</math> (since <math>p^2 (1-p)^3 &gt; 0</math>)</p> <p>Hence, <math>\frac{2}{7} &lt; p &lt; \frac{3}{7}</math></p> <p>P(Blue, Red, Yellow)</p> <p><math>= \frac{3}{4+y} \frac{1}{3+y} \frac{2}{2+y} \frac{3!}{3!}</math></p> <p>P(Blue, Blue, Blue)</p> <p><math>= \frac{3}{4+y} \frac{2}{3+y} \frac{1}{2+y}</math></p> <p><math>P(S=3)</math></p> <p><math>= \frac{3}{4+y} \frac{1}{3+y} \frac{2}{2+y} \frac{1}{2+y}</math></p> <p><math>= \frac{6(3y+1)}{(4+y)(3+y)(2+y)}</math></p>	<p>MI: <math>P(w=2) &gt; P(w=3)</math></p> <p>AI</p> <p>MI: For one possibility correct</p> <p>AI</p>										
8 (i)												
(ii)	<p><math>P(S=3) = \frac{7}{20}</math></p> <p><math>\Rightarrow y^3 + 63y^2 - 178y + 48 = 0</math></p> <p><math>y = 2</math> (since <math>y</math> must be a positive integer)</p> <p><math>P(S=1) = \frac{3 \cdot 2 \cdot 1 \cdot 3!}{6 \cdot 5 \cdot 4 \cdot 2!} = \frac{3}{20}</math></p> <p><math>P(S=2) = \frac{3 \cdot 2 \cdot 2 \cdot 3!}{6 \cdot 5 \cdot 4 \cdot 2!} + \frac{1 \cdot 2 \cdot 1 \cdot 3!}{6 \cdot 5 \cdot 4 \cdot 2!} = \frac{7}{20}</math></p> <p><math>P(S=4) = \frac{3 \cdot 2 \cdot 1 \cdot 3!}{6 \cdot 5 \cdot 4 \cdot 2!} = \frac{3}{20}</math></p> <table border="1" data-bbox="1282 649 1380 940"> <tr> <td><math>s</math></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td><math>P(S=s)</math></td> <td><math>\frac{3}{20}</math></td> <td><math>\frac{7}{20}</math></td> <td><math>\frac{7}{20}</math></td> <td><math>\frac{3}{20}</math></td> </tr> </table>	$s$	1	2	3	4	$P(S=s)$	$\frac{3}{20}$	$\frac{7}{20}$	$\frac{7}{20}$	$\frac{3}{20}$	<p>MI</p> <p>AI</p> <p>MI: Method shown for one of the probabilities</p> <p>BI</p> <p>BI</p> <p>BI</p>
$s$	1	2	3	4								
$P(S=s)$	$\frac{3}{20}$	$\frac{7}{20}$	$\frac{7}{20}$	$\frac{3}{20}$								



Qn	Answer	Guidance
9(i)	<p>Let <math>R</math> be the random variable for the radii of the bolts.</p> $R \sim N(0.8, 0.01^2)$ <p>Percentage of bolts with radius between 0.79cm and 0.82cm</p> $= 100 \times P(0.79 < R < 0.82)$ $= 81.9\% \text{ (3s.f.)}$	B1
9(ii)	<p>Let <math>W</math> be the random variable for the inside radii of the washers.</p> $W \sim N(0.81, 0.012^2)$ <p>Let <math>C</math> be the random variable for the inside circumference of the washers.</p> $C = 2\pi W \sim N(2\pi(0.81), 4\pi^2(0.012^2))$ $\therefore C \sim N(1.62\pi, 0.000576\pi^2)$ <p><math>P(C &gt; c) = 0.05</math>  <math>\Rightarrow P(C \leq c) = 0.95</math>  <math>\therefore</math> using GC, <math>c = 5.21\text{cm}</math> (3s.f.)</p>	<p>M1</p> <p>A1: Correct value of <math>E(C)</math> or <math>\text{Var}(C)</math></p>
9(iii)	$W > R \Leftrightarrow W - R > 0$ $W \leq R + 0.04 \Leftrightarrow W - R \leq 0.04$ $W - R \sim N(0.01, 0.000244)$ <p>Probability that the washer and bolt are a good fit</p> $= P(0 < W - R \leq 0.04)$ $= 0.712 \text{ (3 s.f.)}$	<p>M1: Attempt to find <math>E(W - R)</math></p> <p>A1: Correct value of <math>E(W - R)</math></p> <p>A1</p>

9(iv)	<p>Let <math>O</math> be the random variable for the outside radii of the washers.</p> $O \sim N(\mu, \sigma^2)$ $P(O > 1.25) = 0.15$ $P(O \leq 1.25) = 0.85$ $P\left(Z \leq \frac{1.25 - \mu}{\sigma} = 0.85\right)$ $\frac{1.25 - \mu}{\sigma} = 1.03643$ $\Rightarrow 1.25 - \mu = 1.03643\sigma \text{ -----(1)}$ $P(O < 1.15) = 0.25$ $P\left(Z < \frac{1.15 - \mu}{\sigma} = 0.25\right)$ $\frac{1.15 - \mu}{\sigma} = -0.67449$ $\Rightarrow 1.15 - \mu = -0.67449\sigma \text{ -----(2)}$ <p>(1) - (2):</p> $0.1 = (1.03643 + 0.67449)\sigma$ $\Rightarrow \sigma \approx 0.058448 = 0.0584 \text{ (3 s.f.)}$ $\therefore \mu = 1.25 - 1.03643(0.058448)$ $= 1.19 \text{ (3 s.f.)}$	<p>B1: Correct equation (1)</p> <p>B1: Correct equation (2)</p> <p>M1: Attempt to solve, implied by one correct value</p> <p>A1</p>
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Qn	Answer	Guidance
10(i)	The Production Manager should use a 2-tail test because he's looking for a change in either way.	B1
10(ii)	Since the sample size $n = 50$ is large, we can apply Central Limit Theorem to state that the sample mean time taken to manufacture the control panels follows a normal distribution approximately.	B2
10(iii)	<p>Unbiased estimate of the population mean</p> $= \frac{835.7}{50}$ $= 16.714$ <p>Unbiased estimate of the population variance</p> $= \frac{1}{49} \left( 14067.17 - \frac{835.7^2}{50} \right)$ $= 2.026127$ $= 2.03 \text{ (3 s.f.)}$ <p><math>H_0: \mu = 17</math></p> <p><math>H_1: \mu \neq 17</math></p> <p>Level of significance : 10%</p> <p>Under <math>H_0</math>, since <math>n = 50</math> is large, by Central Limit Theorem, <math>\bar{T} \sim N\left(17, \frac{2.026127}{50}\right)</math> approximately.</p> <p>Test statistic:</p> $Z = \frac{\bar{T} - 17}{\sqrt{2.026127/50}} \sim N(0,1) \text{ approximately}$ <p>Using GC, <math>p</math>-value <math>\approx 0.155 (&gt; 0.1)</math>.</p>	<p>B1: For both correct estimates</p> <p>B1: Correct null and alternative hypotheses</p> <p>B1: Correct distribution for sample mean time</p> <p>B1: Correct <math>p</math>-value or <math>z</math>-value = -1.42075</p>

	Thus, we do not reject $H_0$ and conclude that there is insufficient evidence at 10% level of significance to claim that the mean time taken for the manufacture of the control panel is different using the alternative process.	<p>B1: Compare <math>p</math>-value with significance level or <math>z</math>-value with <math>z</math>-critical value = -1.645</p> <p>B1: Conclusion in the context of the question</p>
10(iv)	The alternative process may produce control panels of better quality; or the process may require fewer other resources (other than time)	B1
10(v)	<p><math>H_0: \mu = 17</math></p> <p><math>H_1: \mu &lt; 17</math></p> <p>Level of significance : 10%</p> <p>Under <math>H_0</math>, since <math>n = 50</math> is large, by Central Limit Theorem, <math>\bar{T} \sim N\left(17, \frac{\sigma^2}{40}\right)</math> approximately.</p> <p>Test statistic:</p> $Z = \frac{\bar{T} - 17}{\frac{\sigma}{\sqrt{40}}} \sim N(0,1) \text{ approximately}$ <p>To reject <math>H_0</math>,</p> $\frac{16.7 - 17}{\frac{\sigma}{\sqrt{40}}} \leq -1.28155$ $\Rightarrow \sqrt{\sigma^2} \leq \left( \frac{16.7 - 17}{-1.28155} \right) \sqrt{40}$ $\Rightarrow \sigma^2 \leq 2.19 \text{ (3 s.f.)}$ <p>The Finance manager will conclude that the alternative process takes a shorter time to manufacture the control panels if the population variance is at most 2.19.</p>	<p>B1: 1-tailed test</p> <p>M1: Set up inequality to reject <math>H_0</math>; allow sign error</p> <p>B1: Correct interpretation</p>