1a) • The number of trials (card draws) is not fixed.

• The probability of getting a King in each draw is not the same.

• The event of drawing a King on a specific draw is not independent of drawing a King on other draws.

b) $$P(X=10) = \frac{\binom{48}{9} \times 9! \times \binom{4}{1}}{\binom{52}{10} \times 10!}$$

$$= 0.0424 \text{ (3sf)}$$

2a) • The probability that an apple is sour is the same for each apple.

• Whether an apple is sour is independent of all other apples.

b) $$X \sim B(8, p)$$

$$P(X \leq 1) = P(X=0) + P(X=1)$$
$$= \binom{8}{0} p^0 (1-p)^8 + \binom{8}{1} p^1 (1-p)^7$$
$$= (1-p)^8 + 8p(1-p)^7$$

c) $X \sim B(8, 0.25)$

$$P(<3 \text{ sour} \mid \geq 1 \text{ sour}) = \frac{P(<3 \text{ sour} \cap \geq 1 \text{ sour})}{P(\geq 1 \text{ sour})}$$

$$= \frac{P(X \leq 2) - P(X \leq 0)}{1 - P(X \leq 0)} = 0.643 \text{ (3sf)}$$

3a) Assumption: Mass of man independent from mass of woman.

$X \sim N(77, 9.8^2)$ $\quad X - Y \sim N(15, 208.4)$

$Y \sim N(62, 10.6^2)$

where X, Y mass of man and woman respectively.

$P(|X-Y|<10) = P(-10 < X-Y < 10)$

$= 0.323$ (3sf)

b) Let $W = X_1 + \ldots + X_3 + Y_1 + \ldots + Y_4 \sim N(479, 737.56)$

$P(W \leq 460) = 0.242$ (3sf)

c) $1.1W \sim N(526.9, 892.4476)$

$P(1.1W \leq 460) = 0.0126$ (3sf)

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4) a) Unbiased estimate of pop mean,

$\bar{x} = \frac{-16}{50} + 18$

$= 17.68$

Unbiased estimate of pop variance,

$s^2 = \frac{1}{49}\left(163 - \frac{16^2}{50}\right)$

$= 3.222$

b) Not necessary to make any assumptions.

Since $n=50$ is large, by CLT

the mean length of fish, $\bar{X}$ is normally distributed approximately so no assumptions needed.

$H_0: \mu = 18$ where μ is pop mean length

$H_1: \mu < 18$ of fish.

Under H_0,

test statistic $Z = \dfrac{\bar{X}-18}{\sqrt{\frac{3.22^2}{50}}} \sim N(0,1)$ approx

p-value $= 0.1037 > 0.1 \Rightarrow H_0$ not rejected.

Hence, at 10% level of significance, there is insufficient evidence to conclude whether the scientist's claim is correct.

c) This means that there is a 0.1 probability that our hypothesis test concludes that the scientist's claim is correct that the mean length of fish is less than 18cm, when in fact the mean length of fish is equal to 18cm.

5) Let X be the r.v. of the mass of salt in a bottle.

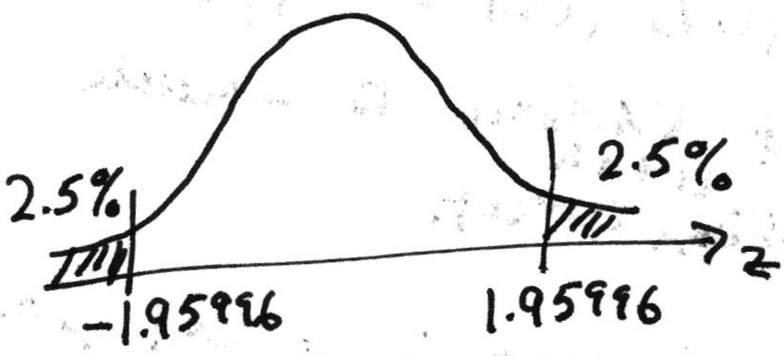
$H_0: \mu = 12$

$H_1: \mu \neq 12$

(a) Since X is known to be normally distributed, $\bar{X}$ is normally distributed so a small sample size is sufficient

(b) Under H_0,

$$Z = \frac{\bar{X} - 12}{\sqrt{\frac{0.8^2}{20}}} \sim N(0,1)$$



Since H_0 rejected,

$$\frac{m-12}{\sqrt{\frac{0.8^2}{20}}} < -1.95996 \quad \text{or} \quad \frac{m-12}{\sqrt{\frac{0.8^2}{20}}} > 1.95996$$

$$m < 11.6 \quad \text{or} \quad m > 11.4 \quad (3sf)$$

Set of values of m: $(0, 11.6) \cup (11.4, \infty)$

6a) The population will be all students in the school.

It will be impractical to send the questionnaire to all the students in the school as there may ~~are~~ be too ~~many~~ many questionnaires to send and process.

b) She could take a sample of 50 students, 25 from J1 and 25 from J2.

This sample should be chosen randomly such that each student in her school has the same ~~chance~~ probability to be selected into her sample (eg by using a random number generator) where each student is chosen independently.

7a) Prob required $= \frac{5!}{7!}$

$$= \frac{1}{42}$$

b) Prob req'd $= P(H \text{ first}) + P(J \text{ last}) - P(H_{\text{first}} \cap J_{\text{last}})$

$$= \frac{6!}{7!} + \frac{6!}{7!} - \frac{1}{42}$$

$$= \frac{11}{42}$$

7c) $P(\text{4 girls next}) = \dfrac{4! \times 4!}{7!}$

$= \dfrac{4}{35}$

d) $P(\text{no two boys next to each other})$

$= \dfrac{4! \times \binom{5}{3} \times 3!}{7!}$

$= \dfrac{2}{7}$