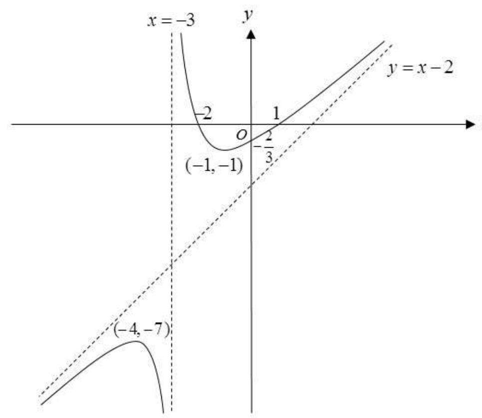
Module 2: Graphing Techniques (Curve Sketching, Functions, Inequalities, Transformation & SOLE)

1. [2015/HCI Promo/10 First Part]

The curve G has equation $y = \dfrac{x^2 - 2x + 2}{x - 1}$. Sketch G, giving the equations of any asymptotes and the coordinates of any stationary points. [4]

2. [2015/DHS/I/5]

The diagram shows the graph of $y = \text{f}(x)$. The graph has a minimum point at $(-1, -1)$ and a maximum point at $(-4, -7)$. It intersects the axes at $x = -2$, $x = 1$ and $y = -\frac{2}{3}$. The equations of the asymptotes are $y = x - 2$ and $x = -3$.

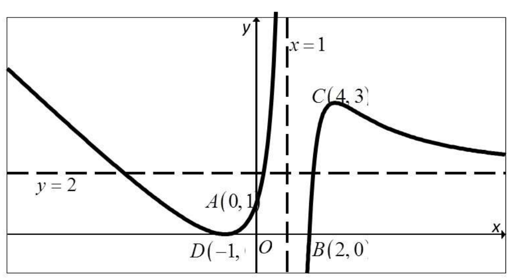


(i) Sketch the graph of $y = \dfrac{1}{\text{f}(x)}$, giving the coordinates of any stationary points, points of intersection with the axes and the equations of any asymptotes. [3]

(ii) Solve the inequality $\text{f}\left(\frac{1}{x}\right) < 0$. [3]

3. [2015/HCI/I/11(a)(i),(ii),(b)]

(a)



The graph of $y = \text{f}(x)$ is shown in the diagram above. It has asymptotes $x = 1$ and $y = 2$. The points A, B, C and D have coordinates $(0, 1)$, $(2, 0)$, $(4, 3)$ and $(-1, 0)$ respectively, with C and D being stationary points.

On separate diagrams, sketch the graphs of

(i) $y = \text{f}(2x - 1)$, [3]

(ii) $y = \text{f}'(x)$, [3]

In each case, state the coordinates of A, B, C and D whenever applicable, and the equations of any asymptotes.

(b) A curve G with equation $y=\dfrac{2x+a}{x^2-b}$, where a and b are constants, has a stationary point at $\left(-4,-\dfrac{1}{4}\right)$ and a vertical asymptote $x=-2$.

(i) Find the values of a and b. [2]

(ii) Find the range of values of x for which G is increasing and is concave downwards. [2]

(iii) By sketching a suitable line on the same diagram as G, find the number of distinct real roots of the equation $5x^3+2x^2-14x+7=0$. [3]

4. **[2014/DHS/I/10]** The function f is defined by $\mathrm{f}:x\mapsto 2+\dfrac{3}{x^2+2x+2},\ x\in\mathbb{R}$.

(i) Sketch the graph of $y=\mathrm{f}(x)$. Show that f does not have an inverse. [2]

(ii) The function f has an inverse if its domain is restricted to $x\geq k$. State the smallest possible value of k. [1]

For the rest of the question, use the new domain found in part **(ii)**.

(iii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iv) Show that f^2 exists and find the range of f^2. [3]

(v) Describe a sequence of two geometrical transformations by which the graph of $y=2+\dfrac{3}{x^2+2x+2}$ can be obtained from the graph of $y=\dfrac{3}{4x^2+4x+2}$. [2]

(vi) By adding a suitable graph to $y=\mathrm{f}(x)$ in part **(i)**, solve the inequality

$$\frac{3}{9x^2+6x+2}<3x-2.$$ [2]

5. **[2014/RVHS/I/11]** The functions f and g are defined by

$$\mathrm{f}:x\mapsto 1+\frac{5}{x^2},\qquad \text{for } x>0,$$

$$\mathrm{g}:x\mapsto \frac{\sin^{-1}x}{x+2},\qquad \text{for } -1\leq x\leq 1.$$

(i) Show that f^{-1} exist and find f^{-1} in a similar form. [4]

(ii) Sketch on the same diagram, the graphs of $\mathrm{f}^{-1}\mathrm{f}$ and $\mathrm{f}\mathrm{f}^{-1}$. Hence solve the equation $\mathrm{f}^{-1}\mathrm{f}(x)=\mathrm{f}\mathrm{f}^{-1}(x)$. [2]

(iii) Explain why the composite function fg does not exist. [1]

(iv) Determine the largest possible domain of g for which fg exists. For this domain of g, find the range of fg in exact form. [4]

6. **[2014/RI Promo/3]** The curve C has equation $\dfrac{(x-1)^2}{9} - y^2 = 1$.

(i) Sketch the curve C, showing clearly the equations of asymptotes, axial intercepts and coordinates of turning points, if any. [3]

(ii) Given that k is a positive constant and C intersects the curve with equation $y^2 + \dfrac{x^2}{k^2} = 1$ at exactly two distinct points, state the range of values of k. [2]

7. **[2014/DHS/I/1]**

(i) A fisherman has 800kg of fish, consisting of mackerel, salmon and tuna.

He may choose to sell all his fish at either Market A or Market B.

The rates offered by the respective markets and his total returns are as follows:

Market	Price (in dollars) per kg			Total Returns (in dollars)
	Mackerel	Salmon	Tuna	
A	7	21	39	20 300
B	5	23	49	23 900

Determine the weight of the salmon that the fisherman has. [3]

(ii) Another fisherman has 600kg of the same types of fish and he claims that he can obtain the exact same total returns as the fisherman in part **(i)** at the respective markets. Determine whether his claim is possible. [1]

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<u>**Answers:**</u>

1. max point $(0,-2)$, min point $(2,2)$

2. $-\dfrac{1}{3} < x < 0$ or $x < -\dfrac{1}{2}$ or $x > 1$

3. $b = 4$, $a = 5$; $(-1,-1)$, $-2 < x < -1$; 3 distinct real roots

4. $f^{-1}(x) = -1 + \sqrt{\dfrac{3}{x-2} - 1}$, $D_{f^{-1}} = (2,5]$; $\left[\dfrac{77}{37}, \dfrac{23}{10}\right)$; $x > 0.753$

5. $f^{-1} : x \mapsto \sqrt{\dfrac{5}{x-1}}$, $x > 1$; $R_{fg} = \left[1 + \dfrac{180}{\pi^2}, \infty\right)$

6. $2 < k < 4$

7. 250 kg Salmon. Not possible.