7. *This question is about asymptotes and range of values of a curve.*

The curve G is given by

$$y = \frac{ax^2 + b}{cx + d},$$

where a, b, c and d are non-zero constants.

(i) Given that $y = 2x + 1$ is an oblique asymptote of G, show that $a + 4d = 0$. [2]

(ii) Given also that $a = 8$, $b = 1$ and $c = 4$,

(a) By first finding the value of d, find the range of values that y can take. [3]

(b) Sketch the graph of G, labelling the equations of asymptote(s), the axial intercept(s) and the stationary point(s), if any. [3]

8. *This question is about the gradient of a curve and curve sketching.*

The curve G is given by

$$y = \frac{4 - x^2}{x^2}, x \in \mathbb{R}, x \neq 0.$$

(i) Find the range of values of x for which G is increasing. [2]

(ii) Sketch the graph of G, labelling the equations of asymptote(s), the axial intercept(s) and the stationary point(s), if any. [3]

Intermediate

1. *This question is about asymptotes and range of values of a curve.*

The curve G is given by

$$y = \frac{ax^2 + bx + c}{x + d},$$

where a, b and c are constants. It is given that $x = -1$ and $y = x + 2$ are asymptotes of C.

(i) Find the values of a, b and d. [3]

(ii) Given G has a stationary point when $x = 1$, find the value of c. [2]

(iii) Find, algebraically, the set of values of y for which there are no points on G. [3]

(iv) Sketch the graph of G, labelling the equations of asymptote(s), the axial intercept(s) and the stationary point(s), if any. [3]

2. *This question is about asymptotes and stationary points of a curve.*

The curve G is given by

$$y = \frac{x + k}{x(x + 3)}, k \neq 0, 3$$

(i) State the equations of the asymptotes of G. [2]

(ii) Find the range of k such that G has two stationary points. [3]

(iii) Given that $k = 5$, sketch the graph of G, labelling the equations of asymptote(s), the axial intercept(s) and the stationary point(s), if any. [3]

3. *This question is about asymptotes and range of values of a curve.*

The curve G is given by

$$y = \frac{x^2 + 6x}{x - 2}.$$

(i) Find the equations of the asymptotes of G. [2]

(ii) Find algebraically the range of values that y cannot take. [3]

At asymptotes, $\dfrac{(y-1)^2}{9} = (x-5)^2.$

$$y = \pm 3(x-5)+1.$$

It is a hyperbola with center (5, 1) and vertices (4, 1) and (6, 1) and asymptotes $y = \pm(3)(x-5)+1$.

(c)

$$4y^2 - 40y - 9x^2 + 18x + 55 = 0.$$
$$4(y^2 - 10y) - 9(x^2 - 2x) + 55 = 0.$$
$$4[(y-5)^2 - 5^2] - 9[(x-1)^2 - 1^2] + 55 = 0.$$
$$4(y-5)^2 - 100 - 9(x-1)^2 + 9 + 55 = 0.$$
$$4(y-5)^2 - 9(x-1)^2 = 36.$$
$$\frac{(y-5)^2}{9} - \frac{(x-1)^2}{4} = 1.$$

At asymptote, $\dfrac{(x-1)^2}{4} = \dfrac{(y-5)^2}{9}$

$$y - 5 = \pm\frac{3}{2}(x-1)$$
$$y = \pm\frac{3}{2}(x-1)+5.$$

It is a hyperbola with center (1, 5) and vertices, (5, – 2) and (5, 4) and asymptotes $y = \pm\dfrac{3}{2}(x-1)+5$.

7. (i) $y = \dfrac{ax^2+b}{cx+d}$

$$\begin{array}{r}
\dfrac{a}{c}x \;-\; \dfrac{ad}{c^2} \\
(x+d)\overline{\big)\; ax^2 \;+\; b \qquad} \\
-\left(ax^2 + \dfrac{ad}{c}x\right) \\
\hline
-\dfrac{ad}{c}x \;+\; b \\
-\left(\dfrac{ad}{c}x + -\dfrac{ad^2}{c^2}\right) \\
\hline
b + \dfrac{ad^2}{c^2}
\end{array}$$

$$\frac{a}{c}x - \frac{ad}{c^2} \equiv 2x+1.$$
$$\frac{a}{c} = 2 \Rightarrow c = \frac{a}{2}$$
$$-\frac{ad}{c^2} = 1.$$
$$ad = -c^2$$
$$ad = -\frac{a^2}{4}$$
$$4ad + a^2 = 0.$$

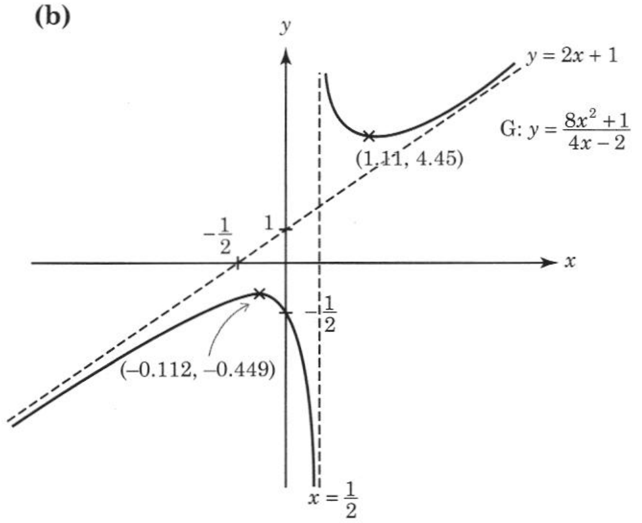
Since $a \neq 0$, $4d + a = 0$.

(ii) (a) $a = 8 \Rightarrow 8 + 4d = 0$

$$d = -2.$$
$$y = \frac{8x^2+1}{4x-2}$$
$$4yx - 2y = 8x^2 + 1$$
$$8x^2 - 4yx + 2y + 1 = 0.$$
$$(-4y)^2 - 4(8)(2y+1) \geq 0.$$
$$16y^2 - 64y - 32 \geq 0.$$
$$y^2 - 4y - 2 \geq 0.$$
$$y = \frac{4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)}$$
$$y = 2 \pm \frac{\sqrt{24}}{2}$$
$$= 2 \pm \sqrt{6}.$$

$$\therefore y \leq 2 - \sqrt{6} \text{ or } y \geq 2 + \sqrt{6}.$$

(b)



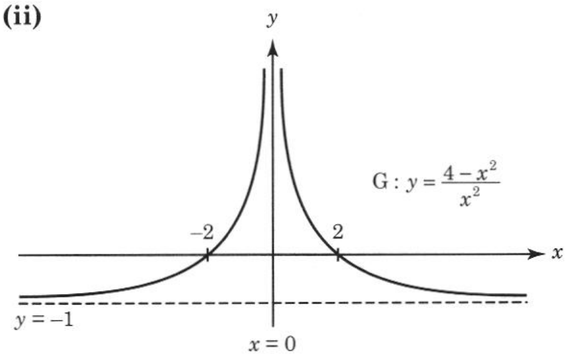
8. (i) $y = \dfrac{4-x^2}{x^2}$

$$y = \frac{4}{x^2} - 1$$
$$\frac{dy}{dx} = -\frac{8}{x^3}$$

let $\dfrac{dy}{dx} \geq 0$,

$$\frac{-8}{x^3} \geq 0.$$
$$\frac{8}{x^3} \leq 0.$$
$$\therefore x < 0.$$

(ii)



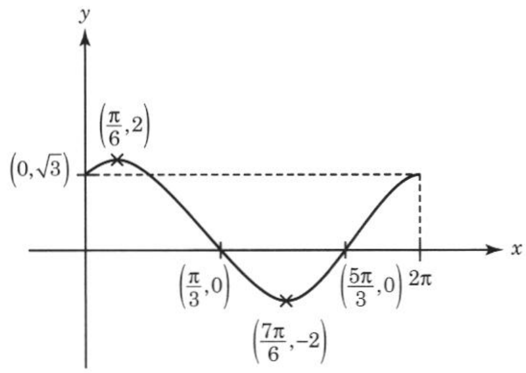
9. (i) $\sin\theta + \sqrt{3}\cos\theta = R\sin(\theta + \alpha)$

$R = \sqrt{1^2 + \sqrt{3}^2} = 2.$

$\alpha = \tan^{-1}\dfrac{\sqrt{3}}{1} = \dfrac{\pi}{3}.$

$\therefore \sin\theta - \sqrt{3}\cos\theta = 2\sin\left(\theta + \dfrac{\pi}{3}\right)$

(ii)



(iii) First, we translate the graph $\frac{\pi}{3}$ units in the negative x-direction.
Then, we stretch the graph by scale factor of 2 units parallel to y-axis.

10. (a) First, we stretch the graph by scale factor of $\frac{2}{3}$ units parallel to y-axis.
Then, we stretch the graph by scale factor of 3 units parallel to x-axis.
Lastly, we translate the graph by 3 units in negative x-direction.

(b) First, we stretch the graph by scale factor of $\frac{3}{2}$ units parallel to x-axis.
Then, we stretch the graph by scale factor of 3 units parallel to y-axis.
Lastly, we translate the graph by 2 units in positive y-direction.

Intermediate

1. (i) $y = \dfrac{ax^2 + bx + c}{x + d}$

$x = -1$ is asymptote $\Rightarrow d = 1.$

$y = x + 2$ is asymptote $\Rightarrow y = x + 2 + \dfrac{e}{x+1}$

$= \dfrac{x^2 + 3x + 2 + e}{x+1}$

$\therefore a = 1$ and $b = 3.$

(ii) $y = x + 2 + \dfrac{e}{x+1}$

$\dfrac{dy}{dx} = 1 - \dfrac{e}{(x+1)^2}$

$\left.\dfrac{dy}{dx}\right|_{x=1} = 1 - \dfrac{e}{(1+1)^2}$

$0 = 1 - \dfrac{1}{4}e.$

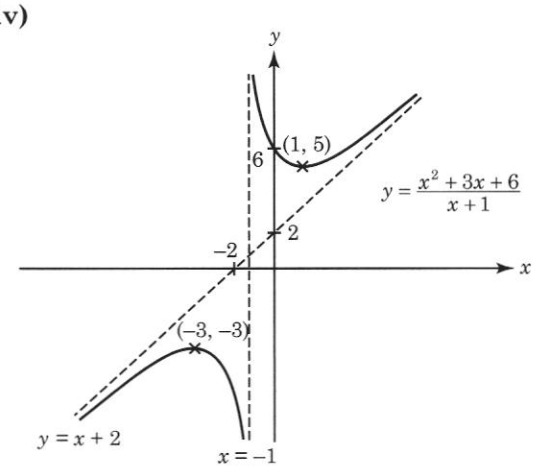
$e = 4$
$c = 2 + e$
$= 6.$

(iii) $y = \dfrac{x^2 + 3x + 6}{x+1}$

$y(x + 1) = x^2 + 3x + 6.$
$x^2 + 3x - yx + 6 - y = 0$
$(3 - y)^2 - 4(1)(6 - y) < 0$
$9 - 6y + y^2 - 24 + 4y < 0$
$y^2 - 2y - 15 < 0$
$(y - 5)(y + 3) < 0$

$-3 < y < 5$
$\therefore \{y \in \mathbb{R}, -3 < y < 5\}.$

(iv)



2. (i) $y = \dfrac{x+k}{x(x+3)}$, $k \neq 0, 3.$

Asymptote: $y = 0$, and
$x = 0$ and $x = -3.$

(ii) $y = \dfrac{x+k}{x(x+3)}$

$\dfrac{dy}{dx} = \dfrac{(1)[x(x+3)] - (x+k)(2x+3)}{[(x(x+3))]^2}$

let $\dfrac{dy}{dx} = 0$, $x(x + 3) - (x + k)(2x + 3) = 0.$

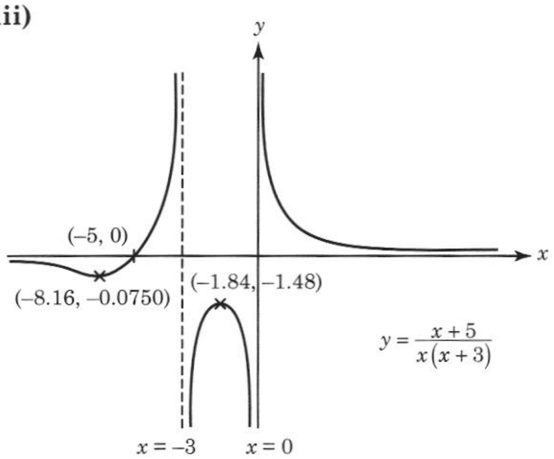
$x^2 + 3x - (2x^2 + 3x + 2kx + 3k) = 0$
$-x^2 - 2kx - 3k = 0$

For G to have two stationary points,

$$(-2k)^2 - 4(-1)(-3k) > 0$$
$$4k^2 - 12k > 0$$
$$k(k-3) > 0$$

$\therefore k < 0$ or $k > 3$.

(iii)



3. (i) $y = \dfrac{x^2 + 6x}{x - 2}$.

$$\begin{array}{r|l} & \;\; x + 8 \\ \hline x - 2 & \;\; x^2 + 6x \\ & -(x^2 - 2x) \\ \hline & \qquad 8x \\ & \quad -(8x - 16) \\ \hline & \qquad\quad 16 \end{array}$$

$$y = x + 8 + \frac{16}{x-2}$$

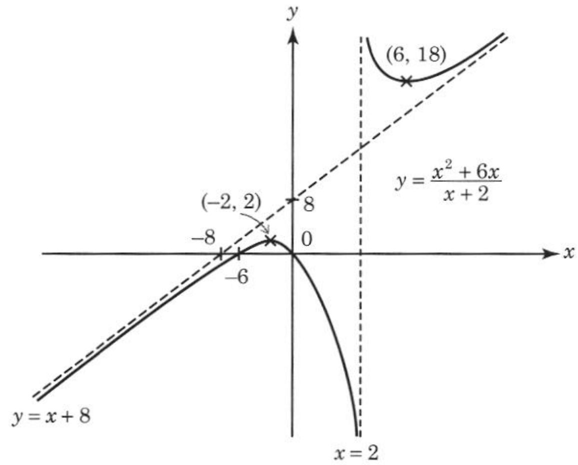
Asymptotes: $y = x + 8$ and $x = 2$.

(ii) $y = \dfrac{x^2 + 6x}{x - 2}$

$$y(x-2) = x^2 + 6x.$$
$$x^2 + 6x - yx + 2y = 0.$$
$$(6 - y)^2 - 4(1)(2y) < 0$$
$$36 - 12y + y^2 - 8y < 0$$
$$y^2 - 20y + 36 < 0$$
$$(y-2)(y-18) < 0$$

$\therefore 2 < y < 18$.

(iii)



(iv) let $k = 1$, then $y = (x - 2) + 10$

$y = x + 8$, which is asymptote of G.

$\therefore k > 1$.

4. (i) $y = \dfrac{2x}{\sqrt{x^2 - 4x + a}}$

$$\frac{dy}{dx} = \frac{2\sqrt{x^2 - 4x + a} - 2x\left(\dfrac{2x-4}{2\sqrt{x^2 - 4x + a}}\right)}{x^2 - 4x + a}$$

let $\dfrac{dy}{dx} = 0$,

$$2\sqrt{x^2 - 4x + a} - 2x\left(\frac{2x-4}{2\sqrt{x^2 - 4x + a}}\right) = 0.$$
$$\sqrt{x^2 - 4x + a} = \frac{x(x-2)}{\sqrt{x^2 - 4x + a}}$$
$$x^2 - 4x + a = x^2 - 2x.$$
$$2x = a$$
$$x = \frac{a}{2}.$$

Since there is only one solution when $\dfrac{dy}{dx} = 0$, G only has one stationary point.

(ii) (a)