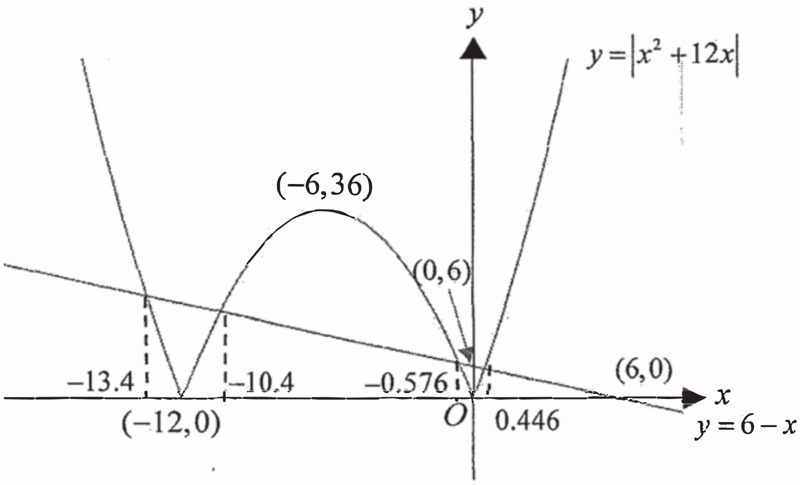
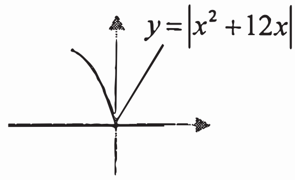
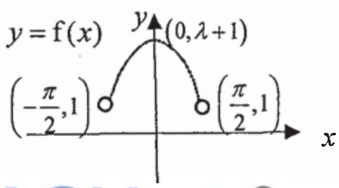
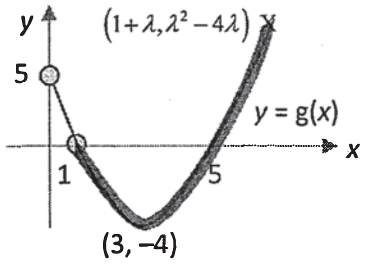
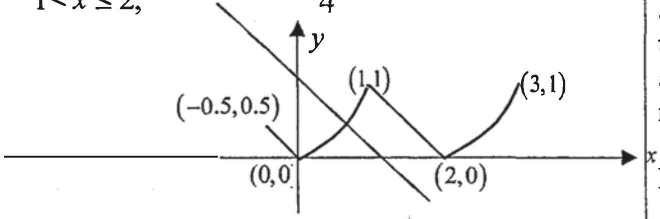
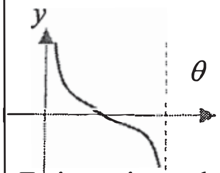
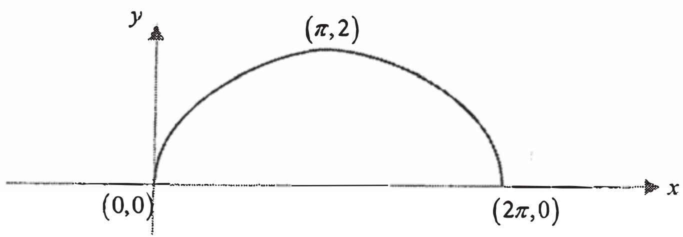
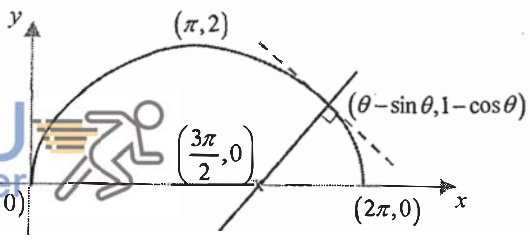
Qn	Solution	Comments
1	Let x, y, and z denote the number of storybooks, Lego sets and sticker sets bought by Ryan respectively. $x=2(z+6)\Rightarrow x-2z=12$ $x+y+z=30$ $14.50x+20.30y+6.10z=513.60$ From GC: $x=14, y=15, z=1$ He bought a total of 14 storybooks, 15 Lego sets, and 1 sticker set.	
2	 $\|x^2+12x\|+x-6>0$ $\|x^2+12x\|>6-x$ From the graphs, $x<-13.4$ or $-10.4<x<-0.576$ or $x>0.446$	Do not blindly follow the G.C. in curve sketching.  The above is a very common wrong answer. Do remember to zoom in/out to check for correctness. The most notable sign that the above is wrong is to see that there are **two** x-intercepts by solving $\|x^2+12x\|=0$ $\|x(x+12)\|=0$ $x=0$ or $x=-12$ How do you think the graph of $y=\|x^2+12x\|$ is related to $y=x^2+12x$? This is a good exercise to think about as well.
3(i)	 Range of f = $(1,\lambda+1]$. Domain of g = $(0,\infty)$ Since range of f $\subseteq$ domain of g , gf exists.	The shape of the graph $y=1+\lambda\cos x$ can be derived from the GC by choosing an suitable value for λ (for e.g. $\lambda=1$). Also, you need to be mindful of the domain (i.e. $D_f=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ where both $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ are excluded)

(ii)	Domain of gf =Domain of f = $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ Range of f = $(1, \lambda+1]$, noting that $\lambda+1>4+1=5$ $\therefore$ range of gf = $\left[-4,(\lambda+1)^2-6(\lambda+1)+5\right]$ $=\left[-4,\lambda^2-4\lambda\right]$ 	It is important to sketch the graph of $y=\mathrm{g}(x)$ when you intend to restrict the domain of g to the range of f.
4a	$\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{2x}\tan(3x)\right]=3e^{2x}\sec^2(3x)+2e^{2x}\tan(3x)$	
b	$\ln\left(xy^2\right)+\sin^{-1}\sqrt{x}=\frac{\pi}{4}$ $\ln x+2\ln y+\sin^{-1}\sqrt{x}=\frac{\pi}{4}$ Differentiate w.r.t x on both sides $\frac{1}{x}+\frac{2}{y}\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{\sqrt{1-x}}\left(\frac{1}{2\sqrt{x}}\right)=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{2x}-\frac{y}{4\sqrt{x}\sqrt{1-x}}$	A commonly seen wrong answer is $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}\sqrt{x}\right)=\frac{1}{\sqrt{1-x}}$. Do not forget to apply chain rule here! Another tip to handle $\ln\left(xy^2\right)$ is to apply Laws of Logarithm to simplify to $\ln x+2\ln y$ before differentiation.
5(i)	$\mathrm{f}(1.5)=2-1.5=0.5$	
(ii)	$\mathrm{f}(x)=\begin{cases}x^2, & 0<x\le 1,\\ 2-x, & 1<x\le 2,\end{cases}$ 	Graphs drawn should be of a reasonable size and use a darker pencil to ensure that your answer could be clearly read. Important features including vertices and endpoints should be label in coordinate form.
(iii)	$\mathrm{f}(x)=-x+\frac{3}{4}$. Consider the line $y=-x+\frac{3}{4}$. $x^2=-x+\frac{3}{4}$ $x^2+x-\frac{3}{4}=0$ $\left(x-\frac{1}{2}\right)\left(x+\frac{3}{2}\right)=0$ $x=\frac{1}{2}$ or $-\frac{3}{2}$	Use the graph in (ii) to help you determine where is the intersection. That will in turn show you which equation to solve to get the answer. Question asks for an exact answer. That means you need to show algebraic workings, so do not

	Since, $0 \le x \le 1$, $x = \frac{1}{2}$	merely use GC to compute the roots of $x^2 + x - \frac{3}{4} = 0$ but show the factorisation. Graph also shows only one intersection point, so you will need to reject one of these 2 answers.
6	Let l cm be the perimeter and A cm^2 be the area of the disc at time t s. $l = \pi r + 2r \Rightarrow \frac{dl}{dr} = \pi + 2$ $A = \frac{1}{2}\pi r^2 \Rightarrow \frac{dA}{dr} = \pi r$ When $l = 40$ and $\frac{dl}{dt} = 3$ $\Rightarrow r = \frac{40}{\pi+2}$ and $\frac{dr}{dt} = \frac{dr}{dl} \times \frac{dl}{dt} = \frac{3}{\pi+2}$ $\Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $= \pi\left(\frac{40}{\pi+2}\right)\left(\frac{3}{\pi+2}\right)$ $= \frac{120\pi}{(\pi+2)^2}$ Hence, the rate of change of the area at $l = 40$ is $\frac{120\pi}{(\pi+2)^2}$ cm^2 / s **Alternatively** Let l cm be the perimeter and A cm^2 be the area of the disc at time t s. $l = \pi r + 2r \Rightarrow \frac{dl}{dt} = (\pi + 2)\frac{dr}{dt}$ $A = \frac{1}{2}\pi r^2 \Rightarrow \frac{dA}{dt} = \pi r \frac{dr}{dt}$ When $l = 40$ and $\frac{dl}{dt} = 3$ $\Rightarrow r = \frac{40}{\pi+2}$ and $\frac{dr}{dt} = \frac{3}{\pi+2}$ $\Rightarrow \frac{dA}{dt} = \pi\left(\frac{40}{\pi+2}\right)\left(\frac{3}{\pi+2}\right)$ $= \frac{120\pi}{(\pi+2)^2}$	A common mistake is to assume that $\frac{dl}{dt} = \frac{dr}{dt}$ which is clear not true. Another common mistake is to write $l = \pi r$ where you only considered the curved part of the semi-circular disc.

	Hence, the rate of change of the area at $l=40$ is $\dfrac{120\pi}{(\pi+2)^2}$ cm^2/s	
7(i)	$u_k = 3r^{k-1}$ $\ln u_k = \ln(3r^{k-1}) = \ln 3 + (k-1)\ln r$ Consider $\ln u_k - \ln u_{k-1} = \left[\ln 3 + (k-1)\ln r\right] - \left[\ln 3 + (k-2)\ln r\right]$ $= (k-1-(k-2))\ln r$ $= \ln r$ Since, r is a constant, $\ln r$ is also a constant. Hence, $\ln u_1, \ln u_2, \ln u_3, \ldots$ is an AP.	Using the difference of the first few consecutive terms to show that sequence is arithmetic is wrong, i.e. $u_2 - u_1 = \ln r$ $u_3 - u_2 = \ln r$ You are merely showing that the first 3 terms form an AP! Using $\ln u_k =$ $\ln 3 + (k-1)\ln r$ and stating that $a = \ln 3$ and $d = \ln r$ is not accepted as well as we are looking for the distinct nature of arithmetic sequences – **any two consecutive terms have a common difference.**
(ii)	$\sum_{k=1}^{30} \ln u_k = 45$ $\frac{30}{2}\left(\ln 3 + \ln\left(3r^{29}\right)\right) = 45$ $\ln\left(9r^{29}\right) = 3$ $9r^{29} = e^3$ $r = \sqrt[29]{\frac{e^3}{9}} = 1.03$ (3 s.f.)	Algebraic errors such as $\ln\left(3r^{29}\right) = 29\ln(3r)$ could be costly.
(iii)	Consider $\dfrac{\frac{1}{u_n}}{\frac{1}{u_{n-1}}} = \dfrac{u_{n-1}}{u_n} = \dfrac{3r^{n-2}}{3r^{n-1}} = \dfrac{1}{r}$. Since $\dfrac{1}{r}$ is a constant, the sequence is geometric. $\dfrac{1}{r} = \dfrac{1}{\sqrt[29]{\frac{e^3}{9}}} = 0.973$ Since $-1 < \dfrac{1}{r} < 1$, hence, this geometric progression is convergent, and $S_\infty = \dfrac{1/3}{1-0.97270} = 12.2$ (3 s.f.)	When applying the formula to find the sum to infinity of a geometric series, ensure you are substituting the correct first term and common ratio. Always use a 5 s.f. or a more accurate answer in your intermediate working.

8i	$x=\theta-\sin\theta \quad \Rightarrow \dfrac{dx}{d\theta}=1-\cos\theta$ $y=1-\cos\theta \quad \Rightarrow \dfrac{dy}{d\theta}=\sin\theta$ $\dfrac{dy}{dx}=\dfrac{\sin\theta}{1-\cos\theta}$ $=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-\left(1-2\sin^2\frac{\theta}{2}\right)}$ $=\dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$ $=\cot\frac{\theta}{2}$ At $\theta=\pi$, $x=\pi$, $y=2$ $\dfrac{dy}{dx}=\cot\frac{\pi}{2}=0$. Hence, the equation of the tangent at $\theta=\pi$ is $y=2$ $\dfrac{dy}{dx}=\cot\frac{\theta}{2}$. As $\theta\to 0$ and $\theta\to 2\pi$, the tangent lines become steeper. Additionally, the coordinates that correspond to $\theta=0$ and $\theta=2\pi$ are $(0,0)$ and $(2\pi,0)$ respectively. Hence, the tangent line at $(0,0)$ and $(2\pi,0)$ are both vertical lines that are parallel to the y-axis.	After obtaining $\dfrac{\sin\theta}{1-\cos\theta}$, use the given answer as a guide to think about the manipulations that you need to do. We need $\dfrac{\theta}{2}$ and $\theta=2\left(\dfrac{\theta}{2}\right)$ so we can consider double angle formula in the MF26.  To investigate the tangents, we can consider the graph of $y=\cot\frac{\theta}{2}$. As $\theta\to 0$ and $\theta\to 2\pi$, $\cot\frac{\theta}{2}\to\infty$ and $\cot\frac{\theta}{2}\to-\infty$ respectively.
ii	 	Relate the sketch of the graph to the properties discussed in part (i). Hence • Tangent at $(\pi,2)$ is horizontal • Tangent at $(0,0)$ and $(2\pi,0)$ is vertical
iii	 The gradient of the line perpendicular to curve C is $-\tan\frac{1}{2}\theta$.	One common error is to carelessly write the gradient of normal to be $\tan\frac{\theta}{2}$ instead.

This line also passes through $\left(\frac{3\pi}{2},0\right)$ and $(\theta-\sin\theta,1-\cos\theta)$. Thus,

$$\frac{(1-\cos\theta)-0}{\left((\theta-\sin\theta)-\frac{3\pi}{2}\right)}=-\tan\tfrac{1}{2}\theta .$$

To solve the above equation for θ, we can consider **two** methods.

Method A (Algebra):

$$1-\cos\theta=-\tan\tfrac{1}{2}\theta\left(\theta-\sin\theta-\frac{3\pi}{2}\right)$$

$$2\sin^2\tfrac{1}{2}\theta=\frac{-\sin\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}\left(\theta-\frac{3\pi}{2}-2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta\right)$$

Since the interception happened in mid-air, $\theta\neq 0$ and $\theta\neq 2\pi$, thus $\sin\frac{1}{2}\theta\neq 0$. Therefore,

$$2\sin\tfrac{1}{2}\theta=\frac{-1}{\cos\frac{1}{2}\theta}\left(\theta-\frac{3\pi}{2}-2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta\right)$$

$$2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta=-\left(\theta-\frac{3\pi}{2}-\sin\theta\right)$$

$$\sin\theta=-\theta+\frac{3\pi}{2}+\sin\theta$$

$$\theta=\frac{3\pi}{2}$$

Coordinate of interception: $\left(\frac{3\pi}{2}-\sin\frac{3\pi}{2},1-\cos\frac{3\pi}{2}\right)=(\frac{3\pi}{2}+1,1)$

Method B (Using GC)

Solve $\frac{(1-\cos\theta)-0}{\left((\theta-\sin\theta)-\frac{3\pi}{2}\right)}=-\tan\frac{1}{2}\theta$ using a GC, we have

$\theta=4.7124$, $\theta=0$ or $\theta=2\pi$. Since interception is in mid-air, $\theta=4.7124$.

Coordinate of interception:
$(4.7124-\sin 4.7124,1-\cos 4.7124)=(5.71,1.00)$ (3 s.f.)

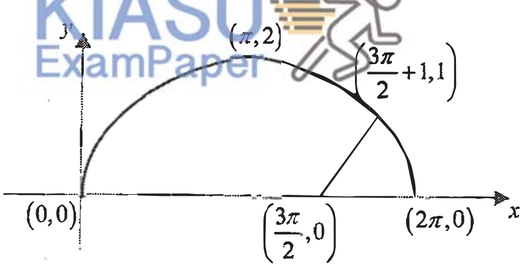
Note that the question did not restrict us in terms of the usage of the GC, so we can solve the equation for θ using GC by plotting the relevant graph. (Ask your friends or tutors to show you the GC steps if you still do not know how to use your GC properly.)

iv

Equation of the path of the missile:

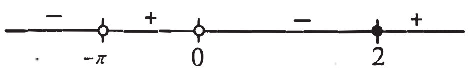
$$y=-\tan\left(\frac{\frac{3\pi}{2}}{2}\right)\left(x-\frac{3\pi}{2}\right)$$

$y=x-\frac{3\pi}{2}$ (or $y=1.00x-4.71$)

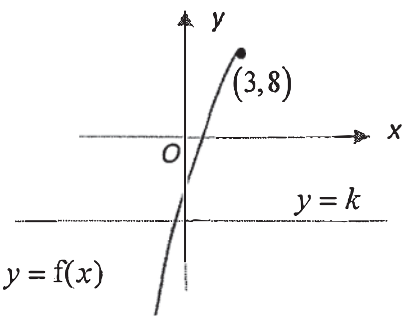


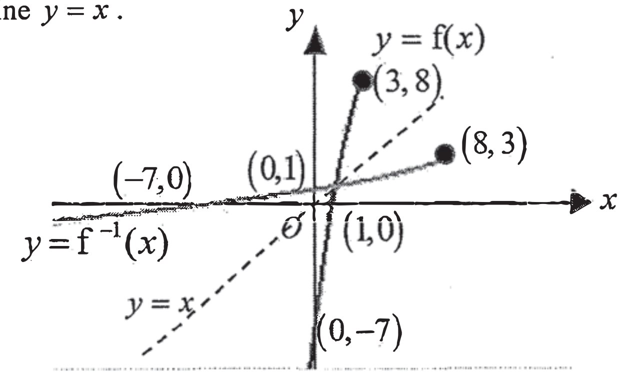
Range of values of x:
$\left[\frac{3\pi}{2}\le x\le\frac{3\pi}{2}+1\right]$

or $\frac{3\pi}{2}\le x\le 5.71$

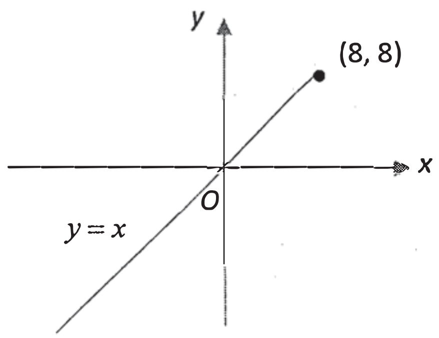
9	$x^3-8=(x-2)(ax^2+bx+c)$ By observation, $a=1$, $c=4$. Compare coefficient of x: $0=4-2b$ $b=2$ $\therefore x^3-8=(x-2)(x^2+2x+4)$ $\begin{cases}a=1\\b=2\\c=4\end{cases}$	
ii	$\dfrac{x^2}{x+\pi}\geq\dfrac{8}{x(x+\pi)}$ $\dfrac{x^3-8}{x(x+\pi)}\geq 0$ $\dfrac{(x-2)(x^2+2x+4)}{x(x+\pi)}\geq 0$ But $x^2+2x+4=(x+1)^2+3>0$ for all real values of x, hence solve: $\dfrac{x-2}{x(x+\pi)}\geq 0$  $\therefore x\geq 2$ or $-\pi<x<0$	There is a **need** to explain why $x^2+2x+4>0$ for all real x. Completing the square is an efficient method to do this.
iii	Replace x with $\ln p$: $\ln p\geq 2$ or $-\pi<\ln p<0$ $\therefore p\geq e^2$ or $e^{-\pi}<p<1$	A common error is to not know how to solve $-\pi<\ln p<0$.
10i	Given $u_n=\dfrac{2}{(n+1)!}$, for $n\geq 1$, $u_n-u_{n+2}=\dfrac{2}{(n+1)!}-\dfrac{2}{(n+3)!}$ $=\dfrac{2(n+2)(n+3)-2}{(n+3)!}$ $=\dfrac{2(n^2+5n+6-1)}{(n+3)!}$ $=\dfrac{2(n^2+5n+5)}{(n+3)!}$	

ii	$\sum_{n=1}^{N}\frac{n^2+5n+5}{(n+3)!}=\frac{1}{2}\sum_{n=1}^{N}\left(u_n-u_{n+2}\right)$ $=\frac{1}{2}\begin{pmatrix} u_1 & - & u_3 \\ +u_2 & - & u_4 \\ +u_3 & - & u_5 \\ & \vdots & \\ +u_{N-2} & - & u_N \\ +u_{N-1} & - & u_{N+1} \\ +u_N & - & u_{N+2} \end{pmatrix}$ $=\frac{1}{2}\left(u_1+u_2-u_{N+1}-u_{N+2}\right)$ $=\frac{1}{2}\left(\frac{2}{2!}+\frac{2}{3!}-\frac{2}{(N+2)!}-\frac{2}{(N+3)!}\right)$ $=\frac{2}{3}-\frac{1}{(N+2)!}-\frac{1}{(N+3)!}$	Follow from (i) and use the symbols u_1, u_2, u_3 ... instead of calculating each value.
iii	Find $\frac{29}{6!}+\frac{41}{7!}+\ \ldots\ +\frac{n^2+5n+5}{(n+3)!}+\ldots\ +\frac{209}{15!}$. Observe that [for $\frac{29}{6!}$, $n=3$ in $\frac{n^2+5n+5}{(n+3)!}$; for $\frac{41}{7!}$, $n=4$ in $\frac{n^2+5n+5}{(n+3)!}$; For $\frac{209}{15!}$, $n=12$ in $\frac{n^2+5n+5}{(n+3)!}$]. Hence, $\frac{29}{6!}+\frac{41}{7!}+\ \ldots+\frac{n^2+5n+5}{(n+3)!}+\ldots\ +\frac{209}{15!}$ $=\sum_{n=3}^{12}\frac{n^2+5n+5}{(n+3)!}$ $=\sum_{n=1}^{12}\frac{n^2+5n+5}{(n+3)!}-\sum_{n=1}^{2}\frac{n^2+5n+5}{(n+3)!}$ $=\frac{2}{3}-\frac{1}{14!}-\frac{1}{15!}-\left(\frac{2}{3}-\frac{1}{4!}-\frac{1}{5!}\right)$ $=0.0500$ (3 s.f.)	

iv	For $\sum_{n=1}^{N}\frac{n^2+5n+5}{(n+3)!}=\frac{2}{3}-\frac{1}{(N+2)!}-\frac{1}{(N+3)!}$, As $N\to\infty$, $\frac{1}{(N+2)!}\to 0$ and $\frac{1}{(N+3)!}\to 0$, $\sum_{n=1}^{N}\frac{n^2+5n+5}{(n+3)!}\to\frac{2}{3}$, a finite value. Hence $\sum_{n=1}^{\infty}\frac{n^2+5n+5}{(n+3)!}$ is convergent.	Compare $\sum_{n=1}^{N}\frac{n^2+5n+5}{(n+3)!}$ with $\sum_{n=1}^{\infty}\frac{n^2+5n+5}{(n+3)!}$, you see that we are discussing convergence where $N\to\infty$. So, let $N\to\infty$ and see if $\sum_{n=1}^{N}\frac{n^2+5n+5}{(n+3)!}$ approaches a finite value and give the reasons why.
v	Replace n with $n+1$: $\sum_{n=2}^{N}\frac{n^2+3n+1}{(n+2)!}=\sum_{n+1=2}^{n+1=N}\frac{(n+1)^2+3(n+1)+1}{(n+1+2)!}$ $=\sum_{n=1}^{N-1}\frac{n^2+5n+5}{(n+3)!}$ $=\frac{2}{3}-\frac{1}{(N+1)!}-\frac{1}{(N+2)!}$	It is easier to approach such questions by starting from $\sum_{n=2}^{N}\frac{n^2+3n+1}{(n+2)!}$ and relating it to $\sum_{n=1}^{N}\frac{n^2+5n+5}{(n+3)!}$. The general rule of thumb is to start from the "new sum" and relate it to "old sum".
11i	Every horizontal line $y=k$ $(k\in\text{range of f})$ cuts the graph $y=\mathrm{f}(x)$ exactly once. Thus f is one-one, and inverse function of f exists. 	Please show the line $y=k$ in your graph.
ii	Let $y=\mathrm{f}(x)$, $x\le 3$ $y=-x^2+8x-7$ $y=-(x-4)^2+9$ $x=4\pm\sqrt{9-y}$ $x\le 3\Rightarrow x=4-\sqrt{9-y}$ $\mathrm{f}^{-1}:x\mapsto 4-\sqrt{9-x},\quad x\in\mathbb{R}, x\le 8$	**Common mistake:** $y=-(x-4)^2+9$ $(x-4)^2=9-y$ $x-4=\sqrt{9-y}$ There is also a need to reject the incorrect expression and give reason.

<table>
<tr>
<td>iii</td>
<td>The graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ are reflections of each other in the line $y=x$.

</td>
<td>Be mindful of the scale used on your axes.

To draw an accurate diagram, you should be using the same scale for both axes.

Question specifically ask for coordinates of axes intersections and endpoints, so make sure you indicate them as such.</td>
</tr>
<tr>
<td>iv</td>
<td>At point of intersection, $\mathrm{f}(x)=\mathrm{f}^{-1}(x)=x$. Hence we solve $\mathrm{f}(x)=x$.
$\mathrm{f}(x)=x$
$\Rightarrow -x^2+8x-7=x$
$\Rightarrow x^2-7x+7=0$
$\Rightarrow x=\dfrac{7\pm\sqrt{49-28}}{2}$
$x\le 3 \Rightarrow x=\dfrac{7-\sqrt{21}}{2}$
Since $y=x$, y-coordinate $=\dfrac{7-\sqrt{21}}{2}$</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>
</td>
<td>Since $D_{\mathrm{ff}^{-1}}=D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}$, the line $y=x$ should be sketched with domain $(-\infty,8]$</td>
</tr>
</table>

12i

Total after first year: $12000(1.04)=12480$

Total after 2nd year: $12000(1.04)^2+12000(1.04)=25459.20$

Hence, the total in his account at the end of 2nd year is $\$25459.20$.

ii

We can tabulate the total in John's account at the start and end of each year:

	Amount of money at the start of the year	Amount of money at the end of the year
1	12000	$12000(1.04)$
2	$12000+12000(1.04)$	$12000(1.04)+12000(1.04)^2$
3	$12000+12000(1.04)+12000(1.04)^2$	$12000(1.04)+12000(1.04)^2+12000(1.04)^3$
n		$12000(1.04)+12000(1.04)^2+12000(1.04)^3+\ldots+12000(1.04)^n$

Total (in dollars) in John's account at the end of n years

$$=12000(1.04)+12000(1.04)^2+\ldots+12000(1.04)^n$$

$$=\frac{12000(1.04)(1-1.04^n)}{1-1.04}$$

$$=312000(1.04^n-1)$$

To write $12000\left[1.04++(1.04)^2+\ldots+(1.04)^n\right]$ before applying the GP formula for a "show" question.

iii

Sub $n=20$, total (in dollars) at the end of 20 years

$$=312000(1.04^{20}-1)$$

$$=371630.42$$

After that, he makes no further deposit, but interests continues to be compounded, hence after another 15 years, total (in dollars) in his account $=371630.42(1.04^{15})=669285.39$

After 35 years, he will have \$669285.39 in his account.

iv

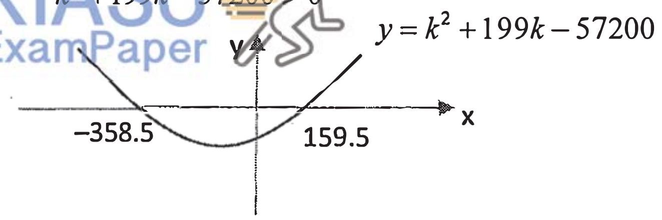
$$2500+2525+2550+\ldots+2500+(k-1)(25)=\frac{k}{2}(5000+25(k-1))$$

$$\frac{k}{2}(5000+25(k-1))>715000$$

$$5000k+25k(k-1)>1430000$$

$$25k^2+4975k-1430000>0$$

$$k^2+199k-57200>0$$



$\Rightarrow k<-358.5$ or $k>159.5$ $\quad\therefore$ Least $k=160$

Hence, he would need 13 years and 4 months.

You should be solving an inequality and not an equation.