## Past Years Examination Questions

**1** It is given that

$$f(x) = 10\cos^2 x - 8\sin x\cos x + 4\sin^2 x.$$

Express $f(x)$ in the form $a\cos 2x + b\sin 2x + c$,

where $a$, $b$ and $c$ are constants. [3]

Hence or otherwise show that the greatest and least values of $f(x)$ are 12 and 2 respectively. [3]

N2002/I/14 *(Either)* (part) (Maths C)

**2** *(a)* (i) Express $7\sin x + 24\cos x$ in the form $R\sin(x + \alpha)$, where $0° < \alpha \leq 90°$.

(ii) Solve the equation $7\sin x + 24\cos x = 12$, for $0° < x < 360°$.

(iii) Find the least positive value of

$$\frac{15}{7\sin x + 24\cos x + 5}$$

and find, for $0° < x < 360°$, the value of $x$ at which this occurs.

*(b)* Given that $\tan x = p$, find an expression, in terms of $p$, for

(i) $\tan\left(\frac{\pi}{4} + x\right)$,

(ii) $\cos 2x$.

Given that $\tan\left(\frac{\pi}{4} + x\right) = \cos 2x$,

(iii) find the possible values of $p$.

N2002/II/4 (AO Maths)

**3** Express $8\sin\theta + 15\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

Find the greatest value, as $\theta$ varies, of

(i) $8\sin\theta + 15\sin\theta$, [1]

(ii) $\dfrac{1}{20 + 8\sin\theta + 15\cos\theta}$, [2]

(iii) $\dfrac{1}{20 + (8\sin\theta + 15\cos\theta)^2}$, [2]

N2003/II/3 (Maths C)

**4** *(a)* (i) Express $7\cos\theta - 9\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(ii) Solve, for $0° < \theta < 180°$, the equation

$$7\cos\theta - 9\sin\theta = 2.$$

*(b)* Given that $\sin x = p$, where $0° < x < 90°$, express in terms of $p$

(i) $\cos x$,

(ii) $\sin 2x$,

Given that $\sin 2x$ can also be expressed as $p + 2p^2$, find

(iii) the value of $p$,

(iv) the angle $x$.

*(c)* Given that $\tan A = 3\tan B$, show that

$$\tan(A - B) = \frac{\sin 2B}{2 - \cos 2B}.$$

N2003/II/4 (AO Maths)

**5** Given that $\sin(A + B) = 2\cos(A - B)$ and $\tan A = {}^1/_3$, find the exact value of $\tan B$.

N2004/I/5*(b)* (Maths C)

**6** The equation $2 - \cos^2\theta = \lambda\cos 2\theta$,

where $\lambda$ is a constant, has a root $\theta = 30°$. Find all the other roots such that $0° \leq \theta \leq 360°$. [4]

N2005/I/1 (Maths C)

**7** By first expanding $\tan(2\theta + \theta)$, show that

$\tan 3\theta = \dfrac{3t - t^3}{1 - 3t^2}$, where $t = \tan\theta$.

Hence solve the equation $4t^3 + 3t^2 - 12t - 1 = 0$.

N2005/I/12 (Maths C)

**8** (i) Show that $\sin 3x + \sin x = 4\sin x\cos^2 x$. [3]

(ii) Find all the angles between 0 and $\pi$ which satisfy the equation

$$\sin 3x + \sin x = 2\cos^2 x.$$ [3]

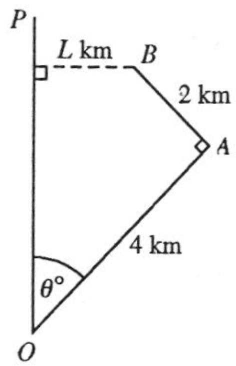
N2008/I/8

**9** (i) Prove the identity $\tan A + \cot A = 2\operatorname{cosec} 2A$. [4]

(ii) Find all the angles between 0° and 360° which satisfy the equation $\tan A + \cot A = 3$. [4]

N2008/II/3

**10** The diagram shows a straight road $OP$. A runner leaves the road at $O$ and runs 4 km in a straight line to a point $A$. She then turns through 90° and runs 2 km in a straight line to a point $B$. The angle $POA$ is $\theta°$, where $0 \leq \theta \leq 90$, and the perpendicular distance of $B$ from the road $OP$ is $L$ km.




(i) Show that $L = 4 \sin \theta - 2 \cos \theta$. [3]

(ii) Express $L$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Find the value of $\theta$ for which $L = 3$. [3]

N2008/II/9

**11** (i) Show that

$$\cos 3x - \cos x = -4 \sin^2 x \cos x. \quad [3]$$

(ii) Hence, or otherwise, solve, for $0 \leq x \leq \pi$ radians, the equation

$$\cos 3x + 2 \cos x = 0. \quad [4]$$

N2009/I/8

**12** $A$ and $B$ are acute angles such that $\sin (A - B) = {}^3\!/_8$ and $\sin A \cos B = {}^5\!/_8$. Without using a calculator, find the value of

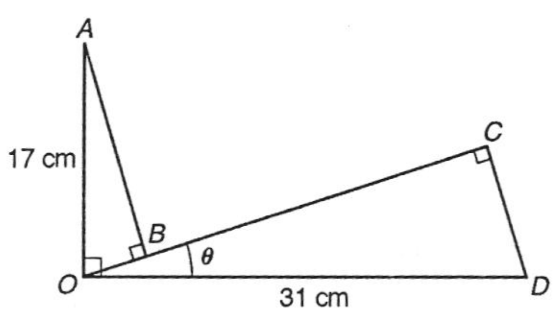
(i) $\cos A \sin B$, [2]

(ii) $\sin (A + B)$, [2]

(iii) $\dfrac{\tan A}{\tan B}$. [2]

N2009/II/1

**13**




The diagram shows three fixed points $O$, $A$ and $D$ such that $OA = 17$ cm, $OD = 31$ cm and angle $AOD = 90°$. The lines $AB$ and $DC$ are perpendicular to the line $OC$ which makes an angle $\theta$ with the line $OD$. The angle $\theta$ can vary in such a way that the point $B$ lies between the points $O$ and $C$.

(i) Show that

$$AB + BC + CD = (48 \cos \theta + 14 \sin \theta) \text{ cm}. \quad [3]$$

(ii) Find the values of $\theta$ for which

$$AB + BC + CD = 49 \text{ cm}. \quad [7]$$

(iii) State the maximum value of $AB + BC + CD$ and the corresponding value of $\theta$. [2]

N2009/II/11