1. **[ASRJC Promos 20 Q7]**

   A curve C has parametric equations

   $$x = 1 - \theta\cos\theta, \quad y = 1 - \cos\theta, \quad \text{for } 0 \leq \theta \leq \pi.$$

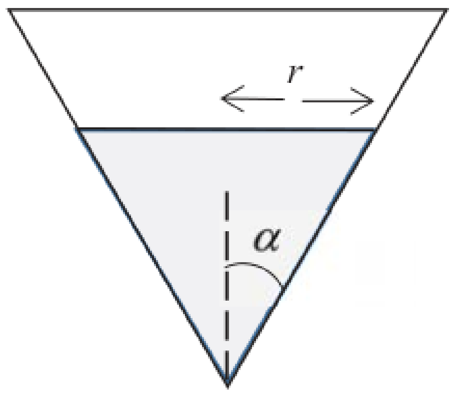
   (a) Find an expression for the gradient of C in terms of θ. What can be said about the tangents to C as $\theta \to 0$ and $\theta \to \pi$? [3]

   (b) Sketch C, showing clearly the features of the curve at the points where $\theta = 0$ and $\theta = \pi$. [2]

   The normal to the curve C where $\theta = \dfrac{\pi}{2}$ cuts the x-axis and the y-axis at Q and R respectively.

   (c) Find the exact area of triangle OQR, where O is the origin. [4]

2. **[ASRJC Promos 20 Q11a]**



   Water leaked out of a container in the form of an inverted cone at a rate of 0.1 m^3 per minute from the bottom of the cone. The semi-vertical angle of the cone is α, where $\tan\alpha = \sqrt{3}$. At time t minutes after the start, the radius of the water surface viewed from the top of the container is given by r m.

   Find the rate of change of the depth of water when the volume of water in the container is 3 m^3. [5]

3. **[CJC Promos 20 Q4]**

   Find the derivative of the following expressions with respect to x, leaving your answers in terms of x only.

   (a) $\ln\sqrt{x^2+1}$, [2]

   (b) $\tan^{-1}\left(\mathrm{e}^{2x}\right)$, [2]

   (c) $x^{\sec 2x}$. [3]

4. **[NYJC Promos 21 Q3]**

Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Point P lies on OA such that $OP = 2PA$ and point Q lies on AB such that $5AQ = 4QB$.

(a) Show that the equation of the line l passing through P and Q can be written as

$$\mathbf{r} = \frac{2}{3}\mathbf{a} + \lambda(4\mathbf{b} - \mathbf{a}), \quad \text{where}\lambda \in \mathbb{R}.$$

[3]

(b) Point X lies on l such that AX is perpendicular to l. If $|\mathbf{a}| = \sqrt{3}$, $|\mathbf{b}| = \frac{1}{2}$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$, find the position vector of X in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

5. **[TJC 19 Promos Q2 (modified)]**

Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, such that $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. The point C lies on the line AB such that

$$\overrightarrow{OC} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}),$$

where $\lambda \in \mathbb{R}$. The area of the triangle OBC is 6 units^2. Given that $\mathbf{a}$ is a unit vector, $|\mathbf{b}| = 4$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $30°$, find the possible position vectors of C in terms of $\mathbf{a}$ and $\mathbf{b}$. [6]

6. **[HCI 21 Promos Q11]**

The points A, B and C have coordinates $(-1, -12, 4)$, $(5, 0, 7)$ and $(6, 1, 4)$ respectively. The line l_1 has equations $\dfrac{x-1}{2} = \dfrac{2-y}{3}, z = 4$ and the line l_2 passes through A and B.

(a) Find the coordinates of the foot of perpendicular from C to l_1. [4]

(b) Find the acute angle between l_1 and l_2. [3]

(c) The point D is on l_2 such that the distance from D to A is twice the distance from D to B. Find the possible point(s) D. [4]

(d) The line l_3 passes through point A and is perpendicular to both l_1 and l_2. Find the equation of l_3. [2]

7. **[CJC Promos 21 Q4]**

The points $A(1, 0, -2)$, $B(3, -1, -2)$ and $C(-3, 7, 0)$ lie on plane p_1. Another plane p_2 has equation $3x - y + 2z = 3$.

(a) Find a vector equation of plane p_1 in the form $\mathbf{r} \cdot \mathbf{n} = d$. [3]

(b) Find the acute angle between p_1 and p_2. [2]

The equation of plane p_3 is given to be $-9x + 3y - 6z = 7$.

(c) Find the shortest distance between p_2 and p_3. [3]

8. **[DHS 21 Promos Q11 (modified)]**

The lengths of the sides of a triangular plot of land are $(x+3)$ m, $(x+3)$ m and $(10-2x)$ m respectively.

(a) Show that the area, A m^2, of the plot of land is given by $A = 4(5-x)\sqrt{x-1}$. [3]

(b) Use differentiation to find the exact maximum value of A, proving that it is a maximum. [4]

9. **[CJC Promos 20 Q7]**

A curve has equation $y = \sqrt{2-xy}$.

(a) Without using a calculator, find the equations of the tangent and normal to the curve at the point P where $x = -1$. [7]

(b) Hence find the exact area of the region bounded by the tangent and normal to the curve at the point P and the y-axis. [2]

10. **[ASRJC 20 Promos Q10i]**

Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\cos^{-1}x\right)$. [2]

11. **[ASRJC 20 Promos Q2a]**

The curve C has equation $y = (x-y)^2$.

By differentiating implicitly twice, show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\left(1 - \dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3$. [4]

Answers

1. (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sin\theta}{\theta\sin\theta - \cos\theta}$.
   The tangents tends to become horizontal lines.

   (c) $\dfrac{(\pi+2)^2}{4\pi}$.

2. 0.0109 m/min.

3. (a) $\frac{x}{x^2+1}$.

   (b) $\frac{2\mathrm{e}^{2x}}{1+\mathrm{e}^{4x}}$.

   (c) $x^{\sec 2x}(\sec 2x)\left(\dfrac{1}{x} + 2(\tan 2x)(\ln x)\right)$.

4. $\overrightarrow{OX} = \frac{1}{21}(17\mathbf{a} - 12\mathbf{b})$.

5. $\overrightarrow{OC} = 6\mathbf{a} - 5\mathbf{b}$ or $\overrightarrow{OC} = 7\mathbf{b} - 6\mathbf{a}$.

6. (a) $F(3, -1, 4)$.

   (b) $61.0°$.

   (c) $D(3, -4, 6)$ or $D(11, 12, 10)$.

   (d) $\mathbf{r} = \begin{pmatrix} -1 \\ -12 \\ 4 \end{pmatrix} + \nu \begin{pmatrix} -3 \\ -2 \\ 14 \end{pmatrix},\ \nu \in \mathbb{R}$.

7. (a) $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = 11$.

   (b) $64.0°$.

   (c) 1.43 units.

8. $\frac{64\sqrt{3}}{9}$.

9. (a) $y = -\frac{2}{3}x + \frac{4}{3}$.
   $y = \frac{3}{2}x + \frac{7}{2}$.

   (b) $\frac{13}{12}$ units^2.

10. $\cos^{-1} x - \dfrac{x}{\sqrt{1-x^2}}$.