MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4038/01

Paper 1

October/November 2008

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

Singapore Examinations and Assessment Board

UNIVERSITY *of* CAMBRIDGE
International Examinations

SPA (KN) T45331/6

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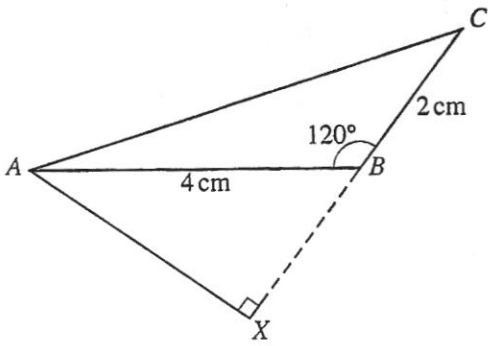
1



The diagram shows a triangle ABC in which $AB = 4\,\text{cm}$, $BC = 2\,\text{cm}$ and angle $ABC = 120°$. The line CB is extended to the point X where angle $AXB = 90°$.

(i) Find the exact length of AX. [2]

(ii) Show that angle $ACB = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$. [3]

2 Solve, for x and y, the simultaneous equations

$$9^x (27)^y = 1,$$
$$8^y \div (\sqrt{2})^x = 16\sqrt{2}.$$ [5]

3 Given that $\mathbf{A} = \begin{pmatrix} 7 & -8 \\ 1 & 6 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and **hence** solve the simultaneous equations

$$8p - 7q + 11 = 0,$$
$$6p + q + 7 = 0.$$ [5]

4 (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}(x^3 \ln x)$. [2]

(ii) Hence find $\int x^2 \ln x \, \mathrm{d}x$. [3]

5 (i) Express $\frac{8x - 46}{(x-5)(x+1)}$ in partial fractions. [3]

(ii) Hence, or otherwise, find the gradient of the curve $y = \frac{8x - 46}{(x-5)(x+1)}$ at the point where $x = 2$. [3]

6 A cyclist starts from rest from a point A and travels in a straight line until he comes to rest at a point B. During the motion, his velocity, $v\,\text{ms}^{-1}$, is given by $v = 6t - \frac{1}{2}t^2$, where t is the time in seconds after leaving A. Find

(i) the time taken for the cyclist to travel from A to B, [1]

(ii) the distance AB, [3]

(iii) the acceleration of the cyclist when $t = 8$. [2]

7 The equation of a curve is $y = \dfrac{\sin x}{2 - \cos x}$. Find the x-coordinate, where $0 < x < \frac{\pi}{2}$, of the point at which the tangent to the curve is parallel to the x-axis. [6]

8 (i) Show that $\sin 3x + \sin x = 4 \sin x \cos^2 x$. [3]

(ii) Find all the angles between 0 and π which satisfy the equation

$$\sin 3x + \sin x = 2 \cos^2 x.$$ [3]

9 Ann is older than her sister Betty. Their ages in years are such that twice the square of Betty's age subtracted from the square of Ann's age gives a number equal to 6 times the difference of their ages. Given also that the sum of their ages is equal to 5 times the difference of their ages, find the age in years of each of the sisters. [6]

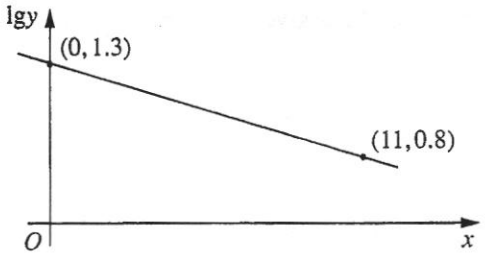
10 (a) Find the smallest value of the integer a for which $ax^2 + 5x + 2$ is positive for all values of x. [3]

(b) Find the smallest value of the integer b for which $-5x^2 + bx - 2$ is negative for all values of x. [3]

11 (i) In the binomial expansion of $\left(x + \dfrac{k}{x}\right)^7$, where k is a positive constant, the coefficients of x^3 and x are the same. Find the value of k. [4]

(ii) Using the value of k found in part (i), find the coefficient of x^7 in the expansion of $(1 - 5x^2)\left(x + \dfrac{k}{x}\right)^7$. [3]

12

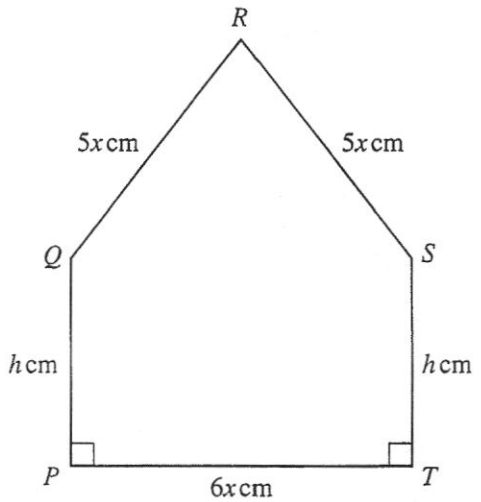


The variables x and y are connected by the equation $y = kb^x$, where k and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points (0, 1.3) and (11, 0.8), obtained by plotting $\lg y$ against x. Estimate

(i) the value, to 2 significant figures, of k and of b, [6]

(ii) the value of y when $x = 8$. [2]

13



The diagram shows a glass window, $PQRST$, consisting of a rectangle $PQST$ of height h cm and width $6x$ cm and an isosceles triangle QRS in which $QR = RS = 5x$ cm. The perimeter of the window is 360 cm.

(i) Show that the area of the window, A cm^2, is given by $A = 1080x - 36x^2$. [4]

Given that x can vary,

(ii) find the stationary value of A, [4]

(iii) determine whether this stationary value is a maximum or a minimum. [1]

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ADDITIONAL MATHEMATICS

4038/02

Paper 2

October/November 2008

2 hours 30 minutes

Additional Materials: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **6** printed pages and **2** blank pages.

Singapore Examinations and Assessment Board

UNIVERSITY of CAMBRIDGE International Examinations

SPA (KN) T45334/5

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1 A man buys a new motorcycle. After t months its value \$$V$ is given by $V = 10000e^{-pt}$, where p is a constant.

(i) Find the value of the motorcycle when the man bought it. [1]

The value of the motorcycle after 12 months is expected to be \$4000. Calculate

(ii) the expected value of the motorcycle after 18 months, [3]

(iii) the age of the motorcycle, to the nearest month, when its expected value will be \$1000. [2]

2 The roots of the quadratic equation $2x^2 - 4x + 3 = 0$ are α and β. Find the quadratic equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$. [7]

3 (i) Prove the identity $\tan A + \cot A = 2 \operatorname{cosec} 2A$. [4]

(ii) Find all the angles between 0° and 360° which satisfy the equation $\tan A + \cot A = 3$. [4]

4 Solve the equation

(i) $2 + \log_3 (3x - 7) = \log_3 (2x - 3)$, [3]

(ii) $3 \log_5 y - \log_y 5 = 2$. [5]

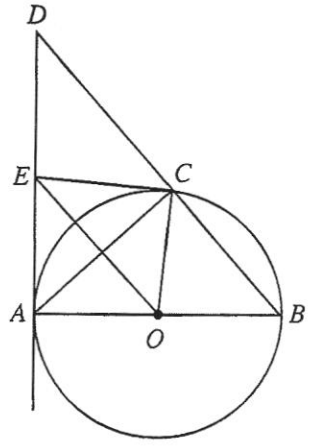
5 The term containing the highest power of x in the polynomial $f(x)$ is $2x^4$. Two of the roots of the equation $f(x) = 0$ are -1 and 2. Given that $x^2 - 3x + 1$ is a quadratic factor of $f(x)$, find

(i) an expression for $f(x)$ in descending powers of x, [5]

(ii) the number of real roots of the equation $f(x) = 0$, justifying your answer, [2]

(iii) the remainder when $f(x)$ is divided by $2x - 1$. [2]

6



The diagram shows a circle, centre O, with diameter AB. The point C lies on the circle. The tangent to the circle at A meets BC extended at D. The tangent to the circle at C meets the line AD at E.

(i) Prove that triangles AEO and CEO are congruent. [4]

(ii) Prove that E is the mid-point of AD. [5]

7 The function f is defined by $f(x) = 4\cos 2x - 2$.

(i) State the amplitude of f. [1]

(ii) State the period of f. [1]

The equation of a curve is $y = 4\cos 2x - 2$ for $0° \leqslant x \leqslant 180°$.

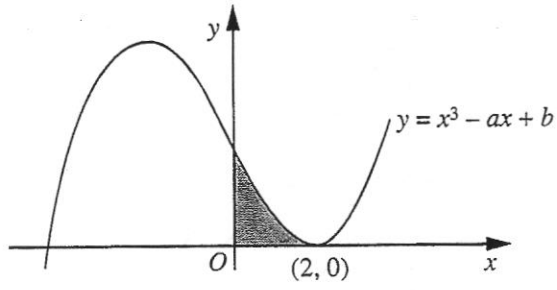
(iii) Find the coordinates of the minimum point of the curve. [1]

(iv) Find the coordinates of the points where the curve meets the x-axis. [3]

(v) Sketch the graph of $y = 4\cos 2x - 2$ for $0° \leqslant x \leqslant 180°$. [2]

(vi) Sketch the graph of $y = |4\cos 2x - 2|$ for $0° \leqslant x \leqslant 180°$. [2]

8



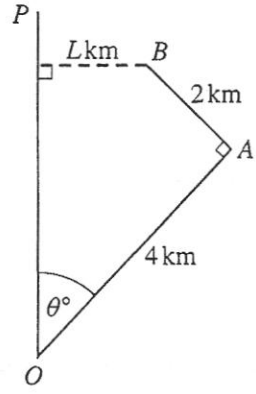
The diagram shows part of the curve $y = x^3 - ax + b$, where a and b are positive constants. The curve has a minimum point at (2, 0). Find

(i) the value of a and of b, [5]

(ii) the coordinates of the maximum point of the curve, [2]

(iii) the area of the shaded region. [3]

9



The diagram shows a straight road OP. A runner leaves the road at O and runs 4 km in a straight line to a point A. She then turns through 90° and runs 2 km in a straight line to a point B. The angle POA is $\theta°$, where $0 \leqslant \theta \leqslant 90$, and the perpendicular distance of B from the road OP is L km.

(i) Show that $L = 4\sin\theta - 2\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Find the value of θ for which $L = 3$. [3]

10 A curve is such that $\frac{dy}{dx} = \frac{6}{(2x-1)^2}$ and $P(2, 9)$ is a point on the curve. The normal to the curve at P meets the y-axis at Q and the x-axis at R.

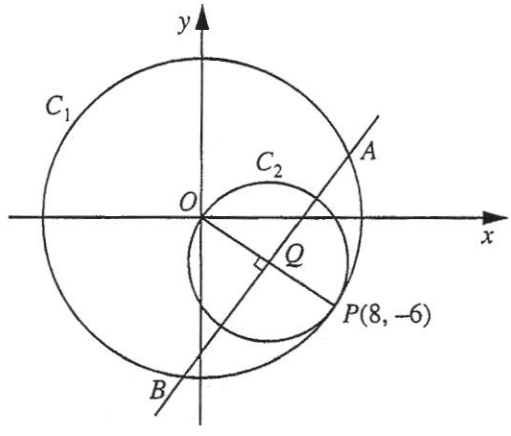
(i) Find the coordinates of the mid-point of QR. [5]

(ii) Find the equation of the curve. [4]

A point (x, y) moves along the curve in such a way that the x-coordinate increases at a constant rate of 0.03 units per second.

(iii) Find the rate of change of the y-coordinate as the point passes through P. [2]

11



The diagram shows two circles C_1 and C_2. Circle C_1 has its centre at the origin O. Circle C_2 passes through O and has its centre at Q. The point $P(8, -6)$ lies on both circles and OP is a diameter of C_2.

(i) Find the equation of C_1. [2]

(ii) Find the equation of C_2. [3]

The line through Q perpendicular to OP meets the circle C_1 at the points A and B.

(iii) Show that the x-coordinates of A and B are $a + b\sqrt{3}$ and $a - b\sqrt{3}$ respectively, where a and b are integers to be found. [7]