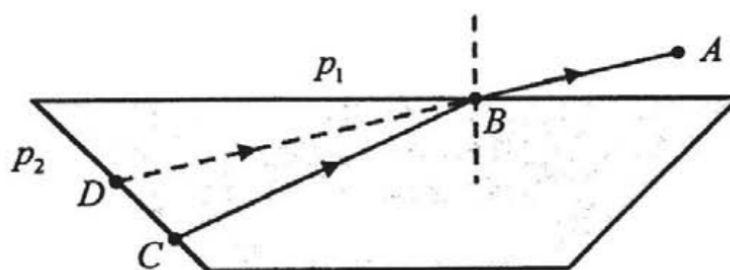


1. [VJC 18 MYE Q2]

The line l has equation $\frac{x-8}{6} = \frac{y}{-1} = \frac{z+1}{-4}$ and $A(8, 0, -1)$ is a point on l . The point B has coordinates $(1, 3, 1)$.

- (a) Find the position vector of the point F on l which is closest to B . [3]
- (b) Explain why l lies in the plane p with equation $x - 2y + 2z = 6$. [2]
- (c) Find the shortest distance from B to p . [2]
- (d) Let \mathbf{n} denote a unit vector perpendicular to plane p . Give the geometrical meaning of $|\overrightarrow{BA} \times \mathbf{n}|$. [1]

2. [VJC 18 MYE Q6]



The diagram shows the vertical cross-section of a slab of glass in the form of a trapezoidal prism, where the top surface is a plane p_1 and the left side of the glass is a plane p_2 . C and D are points in p_2 . The light from a particle placed at C travels in a straight line to B in the glass. The light is refracted at B and travels in a straight line to $A(3, 1, 2)$ in the air. To an observer at A , the particle at C appears to be at $D(0, -7, 4)$.

The plane p_1 has equation $z = 0$ and AB is parallel to $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

- (a) Find the coordinates of B . [3]

\overrightarrow{BC} is in the direction of $-4\mathbf{i} + a\mathbf{j} + \mathbf{k}$, where A is a positive constant. The line BC makes an angle of $\cos^{-1}\left(\frac{1}{9}\right)$ with the normal to p_1 at B .

- (b) Find the value of a . [3]
- (c) Given that the distance BC is 18, show that the position vector of C is

$$-6\mathbf{i} + 19\mathbf{j} + 2\mathbf{k}.$$

The plane p_2 has equation $\mathbf{r} \cdot \mathbf{n} = 34$.

- (d) Given that p_2 is perpendicular to a plane with equation $2x + y = 7$, find \mathbf{n} . [4]

Answers

1. (a) $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(c) 3.

2. (a) $(2, 3, 0)$.

(b) $a = 8$.

(d) $\begin{pmatrix} -1 \\ 2 \\ -5 \end{pmatrix}$.

12. (a) $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$.

(b) 18.4° .

(c) $p = -\frac{2}{3}, q = -\frac{1}{3}$.

(d) $\frac{5\sqrt{2}}{3}$.