Q12	Suggested Solutions	Remarks
(i)	Given: $\overrightarrow{OR}=\begin{pmatrix}6\\4\\0\end{pmatrix}$, $\overrightarrow{OS}=\begin{pmatrix}0\\4\\6\end{pmatrix}$, $\overrightarrow{OT}=\begin{pmatrix}3\\2\\\alpha\end{pmatrix}$.	Label the position vectors of R, S and T correctly.
	$\overrightarrow{SR}=\overrightarrow{OA}=\begin{pmatrix}6\\0\\0\end{pmatrix}$ and $\overrightarrow{ST}=\begin{pmatrix}3\\2\\\alpha\end{pmatrix}-\begin{pmatrix}0\\4\\6\end{pmatrix}=\begin{pmatrix}3\\-2\\\alpha-6\end{pmatrix}$	Use the vectors $\overrightarrow{SR}$, $\overrightarrow{ST}$ to find the normal vector of plane RST
	$\mathbf{n}=\overrightarrow{ST}\times\overrightarrow{SR}=\begin{pmatrix}3\\-2\\\alpha-6\end{pmatrix}\times\begin{pmatrix}6\\0\\0\end{pmatrix}=\begin{pmatrix}0\\6\alpha-36\\12\end{pmatrix}=6\begin{pmatrix}0\\\alpha-6\\2\end{pmatrix}$	Take the dot product of the normal vector and either the position vector of R, S or T to find the constant.
	For equation of plane RST, $\mathbf{r}\cdot\begin{pmatrix}0\\\alpha-6\\2\end{pmatrix}=\begin{pmatrix}0\\4\\6\end{pmatrix}\cdot\begin{pmatrix}0\\\alpha-6\\2\end{pmatrix}=4\alpha-12$	
	For cartesian equation of plane RST, $y(\alpha-6)+2z=4\alpha+12$ $y(\alpha-6)+2z-4\alpha+12=0$ (shown)	Must see the correct normal vector and working to lead to the shown part.
(ii)	We can deduce that Q has coordinates (6, 0, 6). Since P has coordinates, (0, 0, 6), then midpoint of PQ is (3, 0, 6).	Finding mid-point of PQ
	With $\alpha=10$, plane RST has equation $4y+2z=28$. We guess that a point (0, 7, 0) on the plane RST. Distance from point to plane	Guess a point and find the free vector
	$=\left[\begin{pmatrix}0\\7\\0\end{pmatrix}-\begin{pmatrix}3\\0\\6\end{pmatrix}\right]\cdot\frac{\begin{pmatrix}0\\4\\2\end{pmatrix}}{\sqrt{4^2+2^2}}$	Use distance formula
	$=\frac{1}{\sqrt{20}}\left[\begin{pmatrix}-3\\7\\-6\end{pmatrix}\cdot\begin{pmatrix}0\\4\\2\end{pmatrix}\right]=\frac{16}{\sqrt{20}}=\frac{8}{\sqrt{5}}=3.58$ units	
	Alternative method: We can deduce that Q has coordinates (6, 0, 6). Since P has coordinates, (0, 0, 6), then midpoint of PQ is (3, 0, 6).	Finding mid-point of PQ
	With $\alpha=10$, plane RST has equation $4y+2z=28$. A plane containing PQ and parallel to plane RST is	Equation of plane containing PQ and // to RST
	$\mathbf{r}\cdot\begin{pmatrix}0\\10-6\\2\end{pmatrix}=\begin{pmatrix}3\\0\\6\end{pmatrix}\cdot\begin{pmatrix}0\\4\\2\end{pmatrix}=12$	

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Q12	Suggested Solutions	Remarks
	Shortest distance from midpoint of the edge PQ to the plane RST is also distance between 2 parallel planes. $\frac{4(10)-12-12}{\sqrt{(10-6)^2+2^2}}=\frac{16}{\sqrt{4^2+2^2}}=\frac{8}{\sqrt{5}}=3.58$ units	Finding distance between 2 planes
(iii)	normal to the screen is $\mathbf{k}\times\overrightarrow{MN}=\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}3\\-1\\0\end{pmatrix}=\begin{pmatrix}1\\3\\0\end{pmatrix}$. Plane equation of the screen is $\mathbf{r}\cdot\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}3\\4\\6\end{pmatrix}\cdot\begin{pmatrix}1\\3\\0\end{pmatrix}\Rightarrow\mathbf{r}\cdot\begin{pmatrix}1\\3\\0\end{pmatrix}=15$.	Correct plane equation of the screen
	$\overrightarrow{OW}=\begin{pmatrix}3\\3\\2\end{pmatrix}$, $\overrightarrow{OR}=\begin{pmatrix}6\\4\\6\end{pmatrix}\Rightarrow\overrightarrow{WR}=\begin{pmatrix}3\\1\\4\end{pmatrix}$ Vector equation of line for ray of light is $l:\mathbf{r}=\begin{pmatrix}3\\3\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\4\end{pmatrix},\lambda\in\mathbb{R}$	OR Correct equation of the line for ray of light
	We intersect line and plane to find V, $\begin{pmatrix}3+3\lambda\\3+\lambda\\2+4\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\3\\0\end{pmatrix}=15$ $\Rightarrow 3+3\lambda+9+3\lambda=15$ $\Rightarrow\lambda=\frac{1}{2}$ $\overrightarrow{OV}=\begin{pmatrix}3+\frac{3}{2}\\3+\frac{1}{2}\\2+2\end{pmatrix}=\begin{pmatrix}\frac{9}{2}\\\frac{7}{2}\\4\end{pmatrix}$ $\therefore$ Coordinates of V are $\left(\frac{9}{2},\frac{7}{2},4\right)$.	Finding the point of intersection of line and plane $V\left(\frac{9}{2},\frac{7}{2},4\right)$
(iv)	Let $\overrightarrow{OV_1}$ and $\overrightarrow{OV_2}$ be the respective position vectors of the top and bottom end of the screen illuminated by the laser beam.	

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Q12	Suggested Solutions	Remarks
	Since the oscillation is in a vertical direction, $\overrightarrow{OV_1} = \begin{pmatrix} \frac{9}{2} \\ \frac{7}{2} \\ 6 \end{pmatrix}$ and $\overrightarrow{OV_2} = \begin{pmatrix} \frac{9}{2} \\ \frac{7}{2} \\ 0 \end{pmatrix}$	
	$\Rightarrow \overrightarrow{WV_1} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 4 \end{pmatrix}$ and $\overrightarrow{WV_2} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ -2 \end{pmatrix}$	Find $\overrightarrow{WV_1}$ and $\overrightarrow{WV_2}$
	Let θ be the maximum angle of projection of the laser emitted from the projector. $\cos\theta = \dfrac{\overrightarrow{WV_1} \cdot \overrightarrow{WV_2}}{\lvert\overrightarrow{WV_1}\rvert \lvert\overrightarrow{WV_2}\rvert}$	Apply formula for finding angle between $\overrightarrow{WV_1}$ and $\overrightarrow{WV_2}$
	$\cos\theta = \dfrac{\begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 4 \end{pmatrix} \cdot \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ -2 \end{pmatrix}}{\sqrt{\frac{9}{4}+\frac{1}{4}+16}\sqrt{\frac{9}{4}+\frac{1}{4}+4}} = \dfrac{-22}{\sqrt{1924}}$	$\theta = 120.1°$ (1 d.p) *degree answer to 3 s.f. not accepted.
	$\theta = 120.1°$ (1 d.p) or 2.10 radians (3 s.f.)	
	Alternative method: From W to screen, foot of perpendicular F is (4.5, 3.5, 2)	
	Distance $WF = \left\lvert \begin{pmatrix} 4.5 \\ 3.5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix} 1.5 \\ 0.5 \\ 0 \end{pmatrix} \right\rvert$ $= \sqrt{1.5^2 + 0.5^2} = \sqrt{2.5}$	
	From F to top of screen and bottom of screen is 4 and 2 units respectively. $\theta = \tan^{-1}\dfrac{4}{\sqrt{2.5}} + \tan^{-1}\dfrac{2}{\sqrt{2.5}}$ $= 120.1°$ or 2.10 rad	

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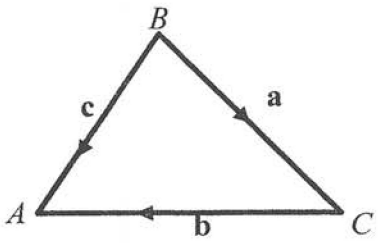
Question 6
Level of Difficulty: Medium
General Comments:

- In part **(a)**, students need to write down $|\mathbf{a}\times\mathbf{c}| = |\mathbf{b}\times\mathbf{c}|$. The use of area of triangle = $\frac{1}{2}ab\sin C$ to prove sine rule is not accepted.
- Parts **(b)(ii)** and **(ii)** are generally well done.
- In part **(iii)**, the approach is to first find the foot of the perpendicular, from P to π, or Q to line passing through X and parallel to normal to plane. Then use ratio theorem to find a point on line representing reflected beam. Students should make use of point X found in part **(ii)**, which is a point on line representing reflected beam.

Qn	Solution
6	**Vectors**
(a)	$|\mathbf{a}\times\mathbf{c}|$ and $|\mathbf{b}\times\mathbf{c}|$ is twice the area of triangle ABC.



$$|\mathbf{a}\times\mathbf{c}| = |\mathbf{b}\times\mathbf{c}|$$

Since $0 < \angle ABC < 180^\circ$ and $0 < \angle BAC < 180^\circ$,

$$|\mathbf{a}||\mathbf{c}|\sin\angle ABC = |\mathbf{b}||\mathbf{c}|\sin\angle BAC$$

$$\frac{\sin\angle ABC}{|\mathbf{b}|} = \frac{\sin\angle BAC}{|\mathbf{a}|}$$

$$\frac{\sin\angle ABC}{AC} = \frac{\sin\angle BAC}{BC} \quad \text{(shown)}$$

(b)(i)

$$\overrightarrow{OP} = \begin{pmatrix}1\\2\\5\end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix}2\\3\\-1\end{pmatrix}, \overrightarrow{PQ} = \begin{pmatrix}1\\1\\-6\end{pmatrix}.$$

$$l_{PQ}: \mathbf{r} = \begin{pmatrix}1\\2\\5\end{pmatrix} + \alpha\begin{pmatrix}1\\1\\-6\end{pmatrix} \text{ where } \alpha \in \mathbb{R}.$$

Let π be the plane for the force shield.

$$\pi: \mathbf{r}\cdot\begin{pmatrix}10\\-3\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\8\end{pmatrix}\cdot\begin{pmatrix}10\\-3\\0\end{pmatrix} = -10$$

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Concept: Angle between line and plane

Use $\sin\theta = \dfrac{|\underset{\sim}{n}\cdot\underset{\sim}{d}|}{|\underset{\sim}{n}||\underset{\sim}{d}|}$

Answer must be in 1 d.p.

$$\text{Required angle} = \sin^{-1}\frac{\left|\begin{pmatrix}1\\1\\-6\end{pmatrix}\cdot\begin{pmatrix}10\\-3\\0\end{pmatrix}\right|}{\sqrt{38}\sqrt{109}} = 6.2^\circ \text{ (1 d.p)}$$

(ii) Let π be the plane for the force shield.

$$\pi: \mathbf{r}\cdot\begin{pmatrix}10\\-3\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\8\end{pmatrix}\cdot\begin{pmatrix}10\\-3\\0\end{pmatrix} = -10$$

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$$\left[\begin{pmatrix}1\\2\\5\end{pmatrix}+\alpha\begin{pmatrix}1\\1\\-6\end{pmatrix}\right].\begin{pmatrix}10\\-3\\0\end{pmatrix}=-10$$

$(10-6)+\alpha(10-3)=-10$

$\alpha=-2$

$$\begin{pmatrix}1\\2\\5\end{pmatrix}+(-2)\begin{pmatrix}1\\1\\-6\end{pmatrix}=\begin{pmatrix}-1\\0\\17\end{pmatrix}$$

$\therefore X$ is $(-1,0,17)$

(iii) Let N be the foot of the perpendicular from P to π.

$$l_{PN}:\mathbf{r}=\begin{pmatrix}1\\2\\5\end{pmatrix}+\beta\begin{pmatrix}10\\-3\\0\end{pmatrix} \text{ where } \beta\in\mathbb{R}.$$

Let $\overrightarrow{ON}=\begin{pmatrix}1\\2\\5\end{pmatrix}+\beta\begin{pmatrix}10\\-3\\0\end{pmatrix}$ for some $\beta\in\mathbb{R}$.

$$\left[\begin{pmatrix}1\\2\\5\end{pmatrix}+\beta\begin{pmatrix}10\\-3\\0\end{pmatrix}\right]\cdot\begin{pmatrix}10\\-3\\0\end{pmatrix}=-10$$

$(10-6)+\beta(100+9)=-10$

$$\beta=-\frac{14}{109}\Rightarrow\overrightarrow{ON}=\begin{pmatrix}1\\2\\5\end{pmatrix}-\frac{14}{109}\begin{pmatrix}10\\-3\\0\end{pmatrix}=\begin{pmatrix}-\frac{31}{109}\\\frac{260}{109}\\5\end{pmatrix}$$

Let the reflection of P in π be P'. $\overrightarrow{ON}=\dfrac{\overrightarrow{OP}+\overrightarrow{OP'}}{2}\Rightarrow\overrightarrow{OP'}=2\overrightarrow{ON}-\overrightarrow{OP}$.

$$\overrightarrow{OP'}=2\begin{pmatrix}-\frac{31}{109}\\\frac{260}{109}\\5\end{pmatrix}-\begin{pmatrix}1\\2\\5\end{pmatrix}=\begin{pmatrix}-\frac{171}{109}\\\frac{302}{109}\\5\end{pmatrix}$$

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$$\overrightarrow{P'X}=\overrightarrow{OX}-\overrightarrow{OP'}=\begin{pmatrix}-1\\0\\17\end{pmatrix}-\begin{pmatrix}-\frac{171}{109}\\\frac{302}{109}\\5\end{pmatrix}=\begin{pmatrix}\frac{62}{109}\\-\frac{302}{109}\\12\end{pmatrix}$$

Equation of reflected beam:

$$\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} \frac{62}{109} \\ -\frac{302}{109} \\ 12 \end{pmatrix} \text{ where } \lambda \in \mathbb{R}$$

Alternative

Let l_{FX} be the line perpendicular to the plane π that passes through X.

Let F be the foot of the perpendicular from Q to l_{FX}.

$$l_{FX} : \mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} + \beta \begin{pmatrix} 10 \\ -3 \\ 0 \end{pmatrix} \text{ where } \beta \in \mathbb{R}.$$

$$\text{Let } \overrightarrow{OF} = \begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} + \beta \begin{pmatrix} 10 \\ -3 \\ 0 \end{pmatrix} \text{ for some } \beta \in \mathbb{R}.$$

Since l_{FX} is perpendicular to QF,

$$\overrightarrow{QF} \cdot \begin{pmatrix} 10 \\ -3 \\ 0 \end{pmatrix} = 0$$

$$\left[\begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} + \beta \begin{pmatrix} 10 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \right] \cdot \begin{pmatrix} 10 \\ -3 \\ 0 \end{pmatrix} = 0$$

$$(-30+9) + \beta(100+9) = 0$$

$$\beta = \frac{21}{109} \Rightarrow \overrightarrow{OF} = \begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} + \frac{21}{109} \begin{pmatrix} 10 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{101}{109} \\ \frac{-63}{109} \\ 17 \end{pmatrix}$$

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Let the reflection of Q in π be Q'. $\overrightarrow{OF} = \dfrac{\overrightarrow{OQ} + \overrightarrow{OQ'}}{2} \Rightarrow \overrightarrow{OQ'} = 2\overrightarrow{OF} - \overrightarrow{OQ}$.

$$\overrightarrow{OQ'} = 2 \begin{pmatrix} \frac{101}{109} \\ \frac{-63}{109} \\ 17 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{16}{109} \\ \frac{-453}{109} \\ 35 \end{pmatrix}$$

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$$\overrightarrow{XQ'} = \overrightarrow{OQ'} - \overrightarrow{OX} = \begin{pmatrix} -\frac{16}{109} \\ \frac{-453}{109} \\ 35 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} = \begin{pmatrix} \frac{93}{109} \\ -\frac{453}{109} \\ 18 \end{pmatrix}$$

Equation of reflected beam:

$$\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} \frac{93}{109} \\ -\frac{453}{109} \\ 18 \end{pmatrix} \text{ where } \lambda \in \mathbb{R}$$

Question 7
Level of Difficulty: Low
General Comments:

- Generally well done for (i).
- (ii) uses the grouping method to ensure that PER appears in the order
- (iii) uses the slotting method to separate the letters P,E,R
- Always check if the letters are distinct/ identical and apply the relevant formula accordingly.

Qn	Solution
7	**P&C**
(i)	Number of ways $= \frac{13!}{2!2!2!}$ $= 778377600$
(ii)	Number of ways $= \frac{9!}{2!}$ $= 181440$ Group PER as a unit. There are 2 such units. Together with NDICULA, there will be a total of 9 units with PER as 2 identical units, hence we need to divide by 2!. There is no need to permute PER as we need to preserve the order of it.
(iii)	Permute remaining letters NDICULA Choose 6 slots, out of 8 slots, to insert the P,E,R Permute PERPER where each letter is repeated twice Number of ways $= 7! \times \binom{8}{6} \times \frac{6!}{2!2!2!}$ $= 12700800$

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Question 8
Level of Difficulty: Medium
General Comments:

- Students have learnt well from the question in lecture test and more students were able to formulate the probabilities required for this question.

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