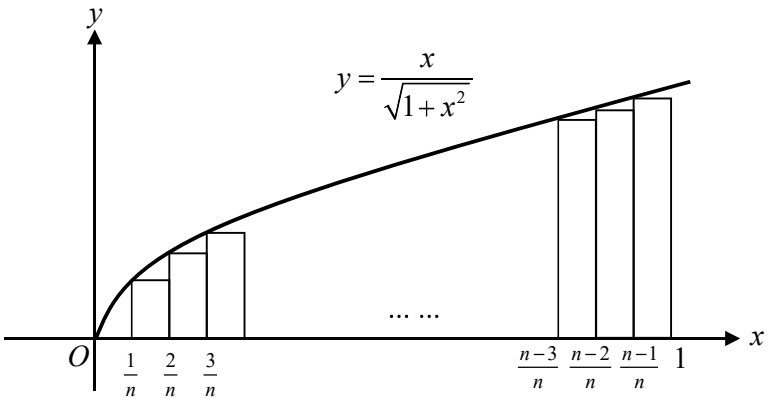
1 The complex numbers z and w satisfy the following equations.

$$2z+1=|w|$$

$$2w-z=4+24\text{i}$$

Find z and w, giving your answers in the form $a+\text{i}b$ where a and b are real numbers. [4]

2 **(a)** The diagram below shows a sketch of the curve $y=\dfrac{x}{\sqrt{1+x^2}}$ for $x\geqslant 0$. Rectangles, each of width $\dfrac{1}{n}$, where $n\in\mathbb{Z}^+$, are drawn under the curve for $0\leqslant x\leqslant 1$.



Show that the total area of all the rectangles, A, can be written as $\dfrac{1}{n}\displaystyle\sum_{r=1}^{n-1}\frac{r}{\sqrt{n^2+r^2}}$. [3]

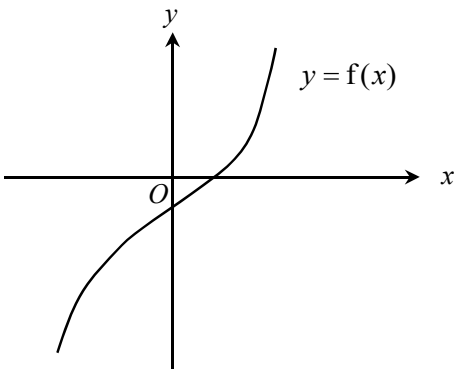
(b) Find the exact value of $\lim\limits_{n\to\infty} A$. [3]

3 The points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively. The points P and Q are fixed and R varies.

(a) Given that $\mathbf{p}$ is non-zero and $(\mathbf{r}-\mathbf{q})\times\mathbf{p}=\mathbf{0}$, find a linear relationship between $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. Describe geometrically the set of all possible positions of the point R. [4]

(b) Given that $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$, $\mathbf{p}=\begin{pmatrix}2\\-5\\3\end{pmatrix}$, $\mathbf{q}=\begin{pmatrix}-4\\1\\2\end{pmatrix}$ and $(\mathbf{r}-\mathbf{q})\cdot\mathbf{p}=0$, find a relationship between x, y and z. Describe geometrically the set of all possible positions of the point R. [4]

4 It is given that $\mathrm{f}(x)$ is a cubic polynomial with real coefficients. The diagram shows the curve with equation $y = \mathrm{f}(x)$.



(a) What can be said about all the roots of the equation $\mathrm{f}(x) = 0$? [2]

Suppose $\mathrm{f}(x) = 2x^3 - 7x^2 + 16x + c$, where c is a real number.

(b) Show that $c = -15$, if the equation $\mathrm{f}(x) = 0$ has a root $x = 1 - 2\mathrm{i}$. [1]

(c) Without using a calculator, determine the other roots of the equation $\mathrm{f}(x) = 0$. [3]

(d) Hence, find the roots of the equation $-15w^3 + 16w^2 - 7w + 2 = 0$. [3]

5 The first four terms of a sequence of numbers are -4, -2, 12 and 38. The sum of the first n terms of this sequence is denoted by S_n.

(a) Explain why S_n cannot be a quadratic polynomial in n. [2]

It is given that S_n is a cubic polynomial.

(b) Find S_n in terms of n. [3]

(c) Show that the nth term of the sequence, u_n is $6n^2 - 16n + 6$. [2]

(d) Hence find $\sum_{n=10}^{2m} (u_n - u_{n-1})$ in terms of m. [3]

6 A curve C has equation $y = \dfrac{ax^2 + bx + c}{x - d}$, where a, b, c and d are constants. It is given that two of its asymptotes are $y = x + 2$ and $x = 1$.

(a) State the value of d, and show that $a = b = 1$. [2]

(b) Using differentiation, find the range of values of c such that the graph of C contains two stationary points. [4]

Use c =14 for the rest of the question.

(c) Sketch C, showing clearly the equations of asymptotes and the coordinates of the turning points. [3]

(d) State the maximum number of roots to the equation

$$k^2(x-5)^2 + \left(\frac{ax^2 + bx + c}{x - d} - 3\right)^2 = k^2, \text{ where } k > 0.$$

Deduce the range of values of k for the maximum number of roots to occur. [2]

7 It is given that

$$\mathrm{f}: x \mapsto \begin{cases} (x-2)^2 & , \quad 0 < x \le 3, \\ 3x - 8 & , \quad 3 < x \le 4, \end{cases}$$

(**a**) **(i)** Sketch the graph of $y = \mathrm{f}(x)$, labelling the coordinates of any turning points and end-points. Explain why f^{-1} does not exist. [3]

(ii) If the domain of f is restricted to $(0, k]$, state the largest value of k such that f^{-1} exists. Hence, for this value of k, find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) The function g is such that

$$\mathrm{g}: x \mapsto \mathrm{e}^x + 3, \quad x \le 0.$$

Find the function fg, giving your answer in similar form. [3]

(b) It is given further that $\mathrm{f}(x) = \mathrm{f}(x + 4)$.

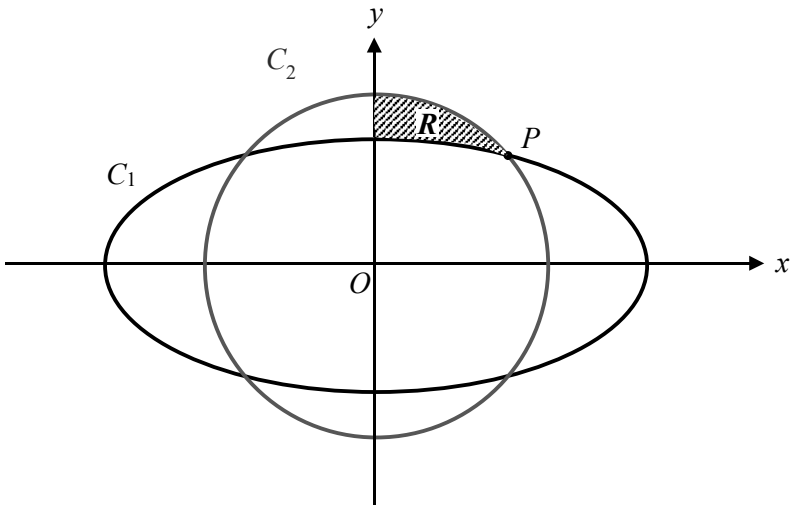
(i) Evaluate $\mathrm{f}(25)$ and $\mathrm{f}(-8)$. [2]

(ii) Sketch the graph of $y = \mathrm{f}\left(\frac{1}{2}x - 1\right)$ for $-8 < x \le 10$. [2]

8 **(a)** Use the substitution $x = \sqrt{15}\sin\theta$ to show that

$$\int \sqrt{15 - x^2}\ \mathrm{d}x = \frac{1}{2}x\sqrt{15 - x^2} + \frac{15}{2}\sin^{-1}\left(\frac{x}{\sqrt{15}}\right) + C\,. \qquad [5]$$

The diagram below shows a sketch of the curves C_1 and C_2.



The curve C_1 has parametric equations

$$x = 6\cos\theta, \qquad y = 2\sqrt{2}\sin\theta$$

for $0 \le \theta \le 2\pi$.

The curve C_2 has equation

$$x^2 + y^2 = 15\,.$$

Given that P is a point of intersection between C_1 and C_2,

(b) determine the exact coordinates of P. [4]

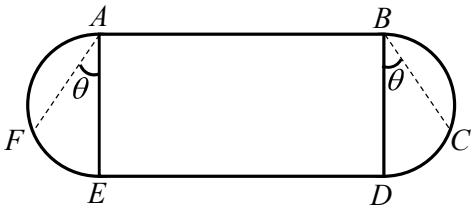
The region R is bounded by curves C_1 and C_2 and the y-axis in the first quadrant.

(c) Show that the area of R is given by

$$m\sin^{-1}\left(n\sqrt{15}\right) - \sqrt{2}\pi$$

where m and n are constants to be determined. [6]

9 The following diagram shows a plot of land formed by two semicircles joined to a rectangle $ABDE$.



Point C lies on the arc BD, with $\angle CBD = \theta$, where $0 < \theta < \frac{\pi}{2}$, and point F lies on the arc AE, with $\angle FAE = \theta$.

As part of a training regime, Nigel runs along the perimeter of the shaded portion $ABCDEF$ as shown in the diagram.

It is given that AE is fixed at $2r$ metres, and AB is twice the length of AE.

(a) Show that the perimeter, P metres, of $ABCDEF$ is

$$P = 4r\left(2 + \cos\theta + \sin\theta\right).$$ [2]

(b) Find the exact value of θ which maximises P and hence find the exact maximum distance that Nigel can run in one round of $ABCDEF$, giving your answer in terms of r. [5]

(c) Nigel plans to run one round with maximum distance at a constant speed of 6 metres per second within 3 minutes. Find the maximum value of r, giving your answer in the form $a + b\sqrt{2}$, where a and b are constants to be determined. [2]

(d) To clearly mark out the shape $ABCDEF$, the management wishes to plant grass within the shape $ABCDEF$. It costs \$0.15 to plant 1 m^2 of grass and the management has a budget of \$10000. Using the value of θ found in part **(b)** and the value of r found in part **(c)**, determine, with justifications, if the management is able to afford to cover the entire shape $ABCDEF$ with grass. [3]

10 A marketing manager of a company wishes to advertise a new product. He has tasked his team to create an engaging video and upload it on InstaFame social platform. He hopes the video would go viral on the internet so that the product will sell well.

According to some analysts, a video is considered to have gone viral when it gets at least a total of 5 million views at the end of the seventh day after its initial posting.

The video is uploaded at the start of a particular day and the number of daily views at the end of the first day is 1196. On each subsequent day, the number of daily views at the end of the day will be three times that of the previous day.

(a) Find the number of daily views at the end of the third day. [2]

(b) Determine if the video will go viral. [2]

The marketing manager also looks at the number of comments being posted on the InstaFame social platform. On each subsequent day, the number of daily comments at the end of the day will be 780 more than that of the previous day. It is given that the number of daily comments posted at the end of the first day is 576.

(c) Find the least number of days for the total number of comments to exceed 100 000. [3]

When the total number of comments reaches 100 000, a software in InstaFame social platform will be activated to remove the oldest comments **at the start of the following day**, helping to save storage space. Upon the activation of the software, w comments will be removed at the start of each day and the number of comments at the end of the day is 3% more than the number of comments at the start of the day. The software will remain activated even when the number of comments drops below 100 000 at any one time.

(d) By taking Day 1 as the day which the software starts removing comments, show that the number of comments at the end of Day n is

$$(1.03)^n M - \frac{103w}{3}\left[(1.03)^n - 1\right]$$

where M is a constant to be determined. [3]

(e) Hence find the range of values of w such that all comments are removed by the end of Day 31. Give your answer correct to the nearest integer. [2]

– END –

CATHOLIC JUNIOR COLLEGE

General Certificate of Education Advanced Level

Higher 2

JC2 Preliminary Examination

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

9758/02

Paper 2

13 Sep 2023

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks											/ **100**
Total	7	8	12	13	8	8	10	10	12	12	

This document consists of 3 printed pages, including this cover page.

Section A: Pure Mathematics [40 marks]

1 Without using a calculator, solve the inequality $\dfrac{x^2+2x-5}{x^2-2x}<2$. [4]

Hence, solve the inequality $\dfrac{x^2+2|x|-5}{x^2-2|x|}>2$. [3]

2 It is given that $y=\mathrm{e}^{-x}\sin x+x-1$.

(a) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=k\mathrm{e}^{-x}\cos x$, where k is a constant to be determined. [2]

(b) By further differentiation of this result, find the Maclaurin series for y, up to and including the term in x^3. [3]

(c) By using the result in part **(b)** and standard series from the List of Formulae (MF26), find the expansion of $\dfrac{\mathrm{e}^{-x}\sin x+x-1}{\cos 2x}$ in ascending powers of x, up to and including the term in x^3, giving the coefficients in exact form. [3]

3 Following the popularity of the action role-playing game, Ginseng Impact, three years ago, developers have developed a strategy game, Ginseng Impactful. The number of people who download Ginseng Impactful, P (in thousands), in a particular city, at time t months, can be modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{26}P\left(13-2P\right).$$

There were 2000 people who download Ginseng Impactful when it is launched.

(a) Show that $P=\dfrac{26}{9\mathrm{e}^{-\frac{1}{2}t}+4}$. [6]

(b) Determine, the time taken, in months, for the number of people who download Ginseng Impactful to double since the launch. [2]

(c) Find the number of people that download Ginseng Impactful in the long run. [2]

(d) Hence sketch the graph showing the number of people that download Ginseng Impactful against time. [2]

4 The plane Π_1 and the line l have equations

$$\Pi_1: \mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 3 \quad \text{and} \quad l: \frac{x+2}{3} = \frac{4-y}{2} = z-3$$

respectively.

(a) Find the acute angle between Π_1 and l. [2]

(b) Find the coordinates of the point of intersection between Π_1 and l. [3]

(c) Find the perpendicular distance from $B(10, -4, 7)$ to Π_1. [3]

The plane Π_2 contains l and is perpendicular to Π_1.

(d) Find a cartesian equation of Π_2. [3]

(e) Without using a calculator, find a vector equation of the line which lies in both Π_1 and Π_2. [2]

Section B: Probability and Statistics [60 marks]

5 During the Great Singapore Sale, a certain electronics store organises a lucky draw to attract more customers. The lucky draw is designed as follows:

A circular board is divided into four sectors labelled with numbers 1, 2, 3, 4 and has angles 144°, 108°, 72°, 36° respectively. The board has a spinner pivoted at the centre of the circular board. When a customer spins the spinner, the spinner comes to rest randomly in one of the four sectors.

Every customer who visits the store is allowed to play one round of the lucky draw. In each round of the lucky draw, the customer gets to spin the spinner twice.

The score, X, of the customer is

- the sum of the two numbers if the numbers from the two spins are different,
- three times the number if the numbers from the two spins are the same.

A customer wins a prize if the score, X, is more than 6.

(a) Show that $\mathrm{P}(X = 6) = 0.15$. [2]

(b) Find the probability distribution of X. [3]

(c) Find the probability that a customer scores less than 10, given that the customer wins a prize. [3]

6 The sales manager of a company that sells air conditioning systems presented the data of average daily temperature, t (°C) and sales, s (in hundred units) at a meeting.

Average daily temperature, t (°C)	18	20	23	26	30	32	33	34
Sales, s (in hundred units)	307	366	497	523	565	580	588	596

(a) Draw a scatter diagram of these data and calculate the value of the product moment correlation coefficient between s and t. Comment on whether a linear model would be appropriate, referring both to the scatter diagram and the value of the product moment correlation coefficient found. [3]

The marketing director proposes that the data should be modelled instead by the regression equation $s = a \ln t + b$, where a and b are constants.

(b) Find the values of a and b, giving your answers to 3 decimal places. [1]

(c) Calculate the product moment correlation coefficient between s and $\ln t$. [1]

(d) Using parts **(a)** and **(c)**, explain which is a better model. [1]

(e) Use the model proposed by the marketing director to estimate the number of units sold when the average daily temperature is 38°C and comment on its reliability. [2]

7 In a class of 18 students, there are 12 girls and 6 boys. A chairperson, a vice-chairperson and a secretary are chosen from the 18 students.

(a) Find the number of ways the chairperson, the vice-chairperson and the secretary can be chosen so that

(i) they are all girls, [1]

(ii) there are at least one girl and at least one boy. [3]

The 18 students sit at random in a circle for a lesson.

Find the probability that

(b) the chairperson, the vice-chairperson and the secretary are all separated from one another, [3]

(c) there are exactly 2 girls sitting between each boy. [3]

8 The store manager at CJStore keeps a bin of large number of oranges. It is known that, on average, p % of the oranges are rotten. The oranges are packed into packets of 10 oranges each. The number of rotten oranges in each packet is denoted by the random variable X.

(a) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X follows a binomial distribution.

(b) It is given that $p = 20$.

(i) Find the probability that there are at least two rotten oranges in a randomly chosen packet. [2]

(ii) 100 packets of oranges are sold at a profit of \$2 per packet. The store offers a discount of \$$d$ for any packet of oranges that contains more than 1 rotten orange. By finding the expected number of packets of oranges that contains more than 1 rotten orange, find the range of values of d, correct to 2 decimal places, if the store manager is expecting a net profit. [3]

(c) The store manager wants to ensure that 95% of the packets of oranges contain at most one rotten orange. Write down an equation satisfied by p. Hence find the value of p. [3]

9 In this question, you should state the parameters of any normal distributions you use.

A supermarket sells apples and guavas. The masses, in grams, of the apples and guavas each follows a normal distribution. The means and standard deviations of the masses of the apples and guavas are shown in the following table:

	Mean (g)	**Standard deviation (g)**
Apples	152	28
Guavas	268	43

Assume that the masses of the apples and guavas are independent of one another.

(a) Three apples are randomly chosen. Find the probability that two of the apples each has mass less than 140g and one of the apples has mass more than 170g. [3]

(b) Find the probability that the total mass of five randomly chosen apples is less than the total mass of three randomly chosen guavas. [3]

The supermarket packs apples and guavas into "Family Packs" for sale. A Family Pack contains three apples and two guavas that are randomly selected.

(c) The probability that a randomly chosen Family Pack differs from the mean mass of Family Packs by less than m grams is 95%. Find the value of m. You may assume that the packing material has negligible mass. [3]

(d) Family Packs are sold at \$5 per kg. Find the probability that a randomly chosen Family Pack costs less than \$5.00. [3]

10 Vesla, an electric vehicle (EV) car manufacturer claims that their EV cars have an average travelling distance of 650 km on a single charge. A car reviewer on the MeTube online video sharing platform wants to test if the EV car manufacturer overstated its claim.

The MeTube car reviewer buys a Vesla EV car to conduct his own test drives. He records the travelling distance on a single charge, x km, on 50 different days over a year. The travelling distance on a single charge, x km, is summarised below:

$$\sum(x-650) = -34.39 \qquad \sum(x-650)^2 = 22769.98$$

(a) Calculate unbiased estimates of the population mean and variance of the travelling distance of an EV car on a single charge. [2]

(b) State hypotheses that the MeTube reviewer can use to test if Vesla has overstated its claim about travelling distance on a single charge, defining any symbols you use. Work out the test statistic in this case, and use it to carry out the test at the 5% level of significance, giving your conclusion in the context of the question. [5]

(c) Explain if the MeTube car reviewer needs to apply Central Limit Theorem for the test to be valid. [2]

Another car reviewer on the TokTik online video sharing platform wants to test if the EV car manufacturer's claim is true. He rents a Vesla EV car to conduct his test drives where he records the travelling distance on a single charge. He records a random sample of 6 test drives and calculates the mean travelling distance on a single charge.

(d) Explain whether the TokTik car reviewer should use a 1-tail or a 2-tail test. [1]

(e) State two assumptions that allow the TokTik car reviewer to carry out his test. [2]

– END –

Answer

1	$z = 6,\ \ w = 5 + 12\mathrm{i}$
2	(b) $\sqrt{2} - 1$
3	(a) $\underset{\sim}{r} = \underset{\sim}{q} + \lambda \underset{\sim}{p}$ (b) $2x - 5y + 3z = -7$
4	(c) $x = 1 + 2\mathrm{i},\ \ x = 1 - 2\mathrm{i},\ \ x = -\frac{3}{2}$ (d) $w = \frac{1}{5} - \frac{2}{5}\mathrm{i},\ \ w = \frac{1}{5} + \frac{2}{5}\mathrm{i},\ \ w = \frac{2}{3}$
5	(b) $S_n = 2n^3 - 5n^2 - n$ (d) $24m^2 - 32m - 342$
6	(a) $d = 1$ (b) $c > -2$ (d) 2 roots; $k > 8$
7	(aii) $k = 2;\ \mathrm{f}^{-1}(x) = 2 - \sqrt{x},\ \ 0 \le x < 4$ (aiii) $\mathrm{fg} : x \mapsto 3\left(\mathrm{e}^x + 3\right) - 8,\ \ x \in \mathbb{R},\ \ x \le 0$ (bi) $\mathrm{f}(25) = 1;\ \ \mathrm{f}(-8) = 4$
8	(b) $\left(3,\ \sqrt{6}\right)$ (c) $m = \frac{15}{2}$ and $n = \frac{1}{5}$
9	(b) $\theta = \frac{\pi}{4};\ \ 4r\left(2 + \sqrt{2}\right)$ metres (c) $270 - 135\sqrt{2}$
10	(a) 10764 (c) Least $n = 16$ (e) $w \ge 4991$

Answer

1	$x<0$ or $1<x<2$ or $x>5$; $-5<x<-2$ or $2<x<5$ or $-1<x<1,\ x\neq 0$
2	(a) $k=-2$ (b) $-1+2x-x^2+\frac{x^3}{3}+\ldots$ (c) $-1+2x-3x^2+\frac{13}{3}x^3+\ldots$
3	(b) 2.56 months (c) 6500
4	(a) 19.1° (b) $(4,0,5)$ (c) $\sqrt{6}$ units (d) $x+5y+7z=39$ (e) $\underset{\sim}{r}=\begin{pmatrix}4\\0\\5\end{pmatrix}+\mu\begin{pmatrix}4\\-5\\3\end{pmatrix},\ \mu\in\mathbb{R}$
5	(c) $\frac{8}{9}$
6	(a) $r=0.947$ (b) $a=436.107$; $b=-923.838$ (c) $r=0.968$ (e) 66 254 units
7	(ai) 1320 (aii) 3456 (b) $\frac{91}{136}$ (c) $\frac{1}{6188}$
8	(bi) 0.624 (bii) $0<d<3.20$ (c) $(1-0.01p)^{10}+0.1p(1-0.01p)^9=0.95$; 3.68
9	(a) 0.0871 (b) 0.674 (c) 153 (d) 0.541
10	(a) $\bar{x}=649.3122$; $s^2=464$ (b) $H_0: \mu=650$ and $H_1: \mu<650$ $z_{\text{test}}=-0.226$; Do not reject H_0

Q1 Complex Numbers [NCK]

Assessment Objectives	Solution	Examiner's Feedback
Solving simultaneous equations involving complex numbers	**Method ①:** $2z+1=\lvert w\rvert$ (1) $2w-z=4+24\text{i}$ (2) From (2): $z=2w-4-24\text{i}$ Substitute into (1): $2(2w-4-24\text{i})+1=\lvert w\rvert$ $4w-7-48\text{i}=\lvert w\rvert$ Let $w=a+b\text{i}$ $4(a+b\text{i})-8-24\text{i}+1=\sqrt{a^2+b^2}$ $(4a-7)+(4b-24)\text{i}=\sqrt{a^2+b^2}$ Comparing Imaginary parts, $4b-24=0$ $b=12$ Comparing Real parts, $4a-7=\sqrt{a^2+b^2}$ $4a-7=\sqrt{a^2+12^2}$ ** $(4a-7)^2=a^2+144$ $15a^2-56a-95=0$ $\Rightarrow a=-\frac{19}{15}$ or $a=5$	This question is not well done. In Method 1 (substitution), students should make the variable z the subject (common in both equations) rather than w (w and $\lvert w\rvert$ in both equations) and substitute this into the other equation. In Method 2 (using Cartesian form of both complex numbers), most students did not realize that z is real. Some students made algebraic slips during expansion. Some students squared both sides of equation when they could compare the real and imaginary parts, resulting in tedious expansions. One of the commonly seen mistakes was the treatment of $\lvert w\rvert$. In this question of complex numbers, some students treated $\lvert w\rvert$ as the absolute real function rather than the modulus of a

From **, since $4a-7=$ a positive real number,

when $a=-\frac{19}{15}$, $4a-7=4\left(-\frac{19}{15}\right)-7<0$

$\Rightarrow$ reject $a=-\frac{19}{15}$

$\therefore\ a=5,\ \ b=12,\ \ z=2(5+12\mathrm{i})-4-24\mathrm{i}=6$

$\Rightarrow w=5+12i,\ \ z=6$

complex number. Note that $|w|=|a+b\mathrm{i}|=\sqrt{a^2+b^2}$ (not $|w|=\pm w$).

Another common mistake is the simplification of $|w|^2$. A significant number of students could not recall the property associated with $|w|^2$. Note that $|w|^2=ww^*=(a+b\mathrm{i})(a-b\mathrm{i})$ (not $|w|^2=|a+b\mathrm{i}|^2=(a+b\mathrm{i})^2$).

Students lost 1 mark if they did not explain why the other root was rejected or did not identify the other root (in the case of using graphing to solve the equation).

<u>Some Common Mistakes</u>

× $|w|=\pm w$

× $|w|=|a+b\mathrm{i}|=a^2+b^2$

× $|w|^2=|a+b\mathrm{i}|^2=(a+b\mathrm{i})^2$

<u>Properties of Complex Numbers</u>

- $|w|=|a+b\mathrm{i}|=\sqrt{a^2+b^2}$
- $|w|^2=ww^*=(a+b\mathrm{i})(a-b\mathrm{i})$

Method ②:

$2z+1=|w|$ (1)

$2w-z=4+24\text{i}$ (2)

$2z+1=$ a positive real number $\Rightarrow$ Let $z=x$ and $w=a+b\text{i}$

From (2): $2(a+bi)-x=4+24\text{i}$

Comparing Real and Imaginary parts,

$2a-x=4$

$2b=24\Rightarrow b=12$

From (1): $2x+1=\sqrt{a^2+b^2}$ (3)

Substitute $b=12$ and $x=2a-4$ into (3):

$2(2a-4)+1=\sqrt{a^2+12^2}$

$4a-7=\sqrt{a^2+12^2}$ **

$(4a-7)^2=a^2+144$

$16a^2-56a+49=a^2+144$

$15a^2-56a-95=0$

$\Rightarrow a=-\frac{19}{15}$ or $a=5$

$\Rightarrow x=-\frac{98}{15}$ or $x=6$

	However $2z+1=$ a positive real number, When $x=-\frac{98}{15}$, $2z+1=2\left(-\frac{98}{15}\right)+1<0$ $\Rightarrow$ reject $x=-\frac{98}{15}$ and $a=-\frac{19}{15}$ $\therefore\ x=6,\ \ a=5,\ \ b=12$ $\Rightarrow w=5+12i,\ \ z=6$	

Q2 Definite Integrals [CSL]

Assessment Objectives	Solution	Examiner's Feedback
Find area of rectangles under the curve and simply using sigma notation	**(a)**	This question is generally not well-attempted. As it is a "show" question, candidates must realise they have to be explicit in the progression towards the result, i.e they should display the relevant thought process through their workings and not skip any steps. In this case, what should be displayed is: 1) how to obtain the height of at least first 2 rectangles and last rectangle 2) present it as a sum 3) simplifying the expression and observing the terms make a sequence 4) generalise the sequence and denote the series with a "sigma" notation Many candidates either skip steps or tried to put up smokescreens to somehow arrive at the final result

When $x=\frac{1}{n}$, $y=\frac{\frac{1}{n}}{\sqrt{1+\left(\frac{1}{n}\right)^2}}$

When $x=\frac{2}{n}$, $y=\frac{\frac{2}{n}}{\sqrt{1+\left(\frac{2}{n}\right)^2}}$

… …

When $x=\frac{n-1}{n}$, $y=\frac{\frac{n-1}{n}}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}$

$$A=\frac{1}{n}\left(\frac{\frac{1}{n}}{\sqrt{1+\left(\frac{1}{n}\right)^2}}\right)+\frac{1}{n}\left(\frac{\frac{2}{n}}{\sqrt{1+\left(\frac{2}{n}\right)^2}}\right)+\frac{1}{n}\left(\frac{\frac{3}{n}}{\sqrt{1+\left(\frac{3}{n}\right)^2}}\right)\ldots+\frac{1}{n}\left(\frac{\frac{n-1}{n}}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}\right)$$

$$=\frac{1}{n^2}\left(\frac{1}{\sqrt{1+\left(\frac{1}{n}\right)^2}}+\frac{2}{\sqrt{1+\left(\frac{2}{n}\right)^2}}+\frac{3}{\sqrt{1+\left(\frac{3}{n}\right)^2}}+\ldots+\frac{n-1}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}\right)$$

$$=\frac{1}{n^2}\left(\frac{n}{\sqrt{n^2+1^2}}+\frac{2n}{\sqrt{n^2+2^2}}+\frac{3n}{\sqrt{n^2+3^2}}+\ldots+\frac{(n-1)n}{\sqrt{n^2+(n-1)^2}}\right)$$

$$=\frac{1}{n}\left[\frac{1}{\sqrt{n^2+1^2}}+\frac{2}{\sqrt{n^2+2^2}}+\frac{3}{\sqrt{n^2+3^2}}+\ldots+\frac{(n-1)}{\sqrt{n^2+(n-1)^2}}\right]$$

$$=\frac{1}{n}\sum_{r=1}^{n-1}\frac{r}{\sqrt{n^2+r^2}}\ (\text{shown})$$

Understand the relationship between area of rectangles under the curve and integral	**(b)**	$\lim_{n\to\infty} A = \int_0^1 \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x$ $= \frac{1}{2}\int_0^1 2x\cdot\left(1+x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$ $= \frac{1}{2}\left[\frac{\left(1+x^2\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1$ $= \sqrt{2}-1$	This question is not well attempted. Most candidates did not realise that $\lim_{n\to\infty} A$ simply denotes the area sum of an infinite number of rectangles, which should translate to the **exact area under the curve.** So they went ahead to try to take limits on the expression in (a) to try to find out what it converges to, and fails to do so spectacularly, getting 0 or 1. Candidates are advised to devote care and understanding when doing their revision.

Q3 Vectors [LWY]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Find a relationship between three vectors Describe a line geometrically	**(a)**	$\underset{\sim}{r}-\underset{\sim}{q}$ is parallel to $\underset{\sim}{p}$ $\Rightarrow \underset{\sim}{r}-\underset{\sim}{q}=\lambda\underset{\sim}{p}$ $\underset{\sim}{r}=\underset{\sim}{q}+\lambda\underset{\sim}{p}$ The point R lies on a line that passes through the point Q and is parallel to the vector $\underset{\sim}{p}$.	Presentation of vectors is a prevalent issue here. Many candidates did not bother to include the tilde, rendering the relationship as that of non-vectors, potentially risking it being struck off completely. There are statements that go - $\underset{\sim}{p}$, $\underset{\sim}{q}$, $\underset{\sim}{r}$ are collinear points - $\underset{\sim}{r}$ lies on line…. It gives the impression that the candidate lacks the understanding of the difference between a vector and a point, or simply cannot be bothered to present properly.
Find a cartesian equation of a plane Describe a plane geometrically	**(b)**	$\left(\underset{\sim}{r}-\underset{\sim}{q}\right)\cdot\underset{\sim}{p}=0$ $\underset{\sim}{r}\cdot\underset{\sim}{p}-\underset{\sim}{q}\cdot\underset{\sim}{p}=0$ $\underset{\sim}{r}\cdot\underset{\sim}{p}=\underset{\sim}{q}\cdot\underset{\sim}{p}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}2\\-5\\3\end{pmatrix}=\begin{pmatrix}-4\\1\\2\end{pmatrix}\cdot\begin{pmatrix}2\\-5\\3\end{pmatrix}$ $2x-5y+3z=-7$ The point R lies on a plane that contains the point Q and is perpendicular to the vector $\underset{\sim}{p}$.	Candidates are advised to be careful when copying over the vectors to work with. There are some candidates with serious misconceptions about the dot product:

			$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 2x \\ -5y \\ 3z \end{pmatrix} = \begin{pmatrix} -8 \\ -5 \\ 6 \end{pmatrix}$ $\begin{pmatrix} 2x+8 \\ -5y+5 \\ 3z-6 \end{pmatrix} = 0$ The second and third line are grave conceptual errors. - Dot products yield scalars. - $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is NOT 0

Q4 Complex Numbers [NCK]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Understanding of Conjugate root Theorem and reasoning	**(a)**	There is only one(positive) real root in the equation $f(x)=0$. Since the equation has all real coefficients, then the two other roots must be a pair of complex conjugates.	Poorly done. From the graph, there is only one positive real root. The other two roots must be a pair of complex conjugates as the coefficients of the polynomial are all real. However, students simply said that there are 2 imaginary roots. It should be **a pair of complex conjugate roots**.
Applying nature of roots	**(b)**	Since $x=1-2\text{i}$ is a root of $2x^3-7x^2+16x+c=0$, $2(-11+2\text{i})-7(-3-4\text{i})+16(1-2\text{i})+c=0$ $15+c=0$ $\therefore c=-15$ (shown)	Well done. A number of students solved **part (c)** first instead of **part (b)**. Students must pay close attention to the number of marks allocated for each part and use the most efficient method to solve.
Procedure of finding complex roots of a polynomial	**(c)**	Since all the coefficients are real, $x=1+2\text{i}$ is another root of $2x^3-7x^2+16x+c=0$. $2x^3-7x^2+16x-15=0$ $[x-(1+2\text{i})][x-(1-2\text{i})](2x-k)=0$ $[(x-1)+2\text{i})][(x-1)-2\text{i})](2x-k)=0$ $[x^2-2x+5](2x-k)=0$ Comparing the coefficient of constant term (or by long division), $-5k=-15 \Rightarrow k=3$ Therefore, the last root is $x=\frac{3}{2}$	Many students have forgotten to mention the most fundamental concept of the conjugate root theorem. They simply assumed that the conjugate root exists and went on to find the third real root. A number of students ignored the "Without the use a calculator" and used the calculator to identify one of

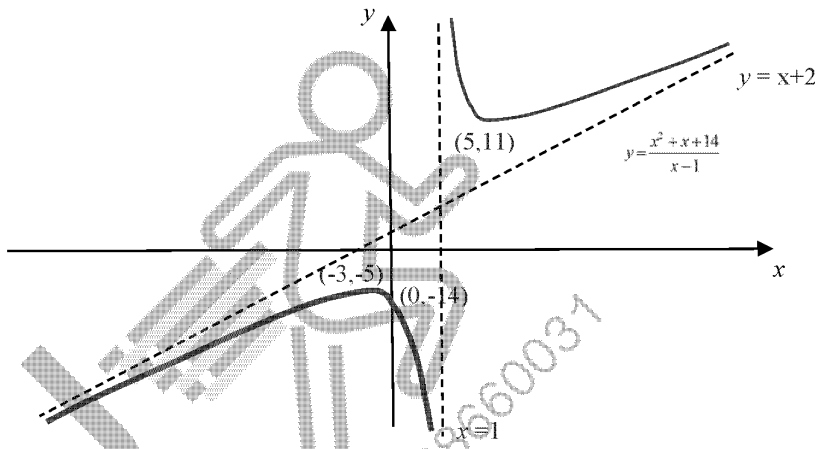
		The roots are $x=1+2\text{i},\ x=1-2\text{i}$, $x=\frac{3}{2}$	the roots as $\frac{3}{2}$. Students who used remainder and factor theorem to identify the third root as $\frac{3}{2}$ were also not awarded full credit. Students were confused over factors and roots. Do not that $2x-3=0$ is a factor and $x=\frac{3}{2}$ is a root.
Deducing roots of a polynomial by establishing relationship with another polynomial	**(d)**	$2x^3-7x^2+16x+c=0$ Replace x with $\frac{1}{w}$ $2\left(\frac{1}{w}\right)^3-7\left(\frac{1}{w}\right)^2+16\left(\frac{1}{w}\right)-15=0$ $2-7w+16w^2-15w^3=0$ Hence, the roots are $\frac{1}{w}=1+2\text{i}$; $\frac{1}{w}=1-2\text{i}$; $\frac{1}{w}=\frac{3}{2}$ $w=\frac{1}{1+2\text{i}}=\frac{1}{5}-\frac{2}{5}\text{i}$ $w=\frac{1}{1-2\text{i}}=\frac{1}{5}+\frac{2}{5}\text{i}$ $w=\frac{2}{3}$	Poorly attempted. Students who simply pressed G.C. to obtain the 3 roots were not given any credit. "Hence" was clearly stated in the question but not adhered to. The relationship, i.e. the replacement should be clearly written in the solution. Students who succeeded in identifying the replacement often left the answer as $w=\frac{1}{1+2\text{i}}$ and $w=\frac{1}{1-2\text{i}}$ without further simplification.

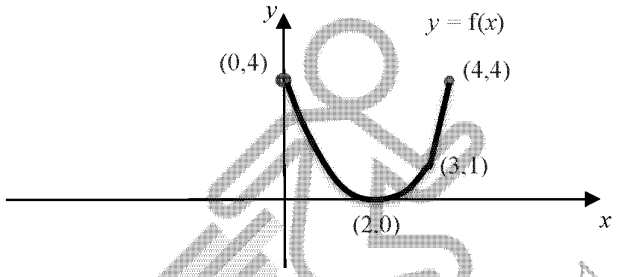
Q5 Sequences and Series [LWY]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Describe the relationship between sum to n terms and the nth term (in terms of degree of the polynomial)	**(a)**	**Method ①:** The nth term, u_n, is always one degree less than S_n since $u_n = S_n - S_{n-1}$. If S_n is quadratic, u_n would be linear but it is not since there is no common difference between consecutive terms. **Method ②: Proof by Contradiction** Suppose $S_n = an^2 + bn + c$ $S_n = an^2 + bn + c$ $a + b + c = -4$ $4a + 2b + c = -4 - 2 = -6$ $9a + 3b + c = -6 + 12 = 6$ $16a + 4b + c = 6 + 38 = 44$ Using G.C., no solution found. Hence S_n cannot be a quadratic polynomial.	Students assume that if S_n is a a quadratic polynomial in n, then it must be an arithmetic progression. This is **not** true. The easiest way to solve would be to prove by contradiction. This is done by first assuming that S_n is a a quadratic polynomial in n, followed by formulation of 4 equations with 4 unknowns and solving them. When no solution is found using a G.C., we can conclude that S_n cannot be a quadratic polynomial. Note that for Method ②, all 4 equations must be formed. If a student used only 3 equations, there will be a solution.
Formulate and solve system of linear equations	**(b)**	$S_n = an^3 + bn^2 + cn + d$ $a + b + c + d = -4$ $8a + 4b + 2c + d = -4 - 2 = -6$ $27a + 9b + 3c + d = -6 + 12 = 6$ $64a + 16b + 4c + d = 6 + 38 = 44$ Using G.C., $a = 2,\ b = -5,\ c = -1,\ d = 0$ $S_n = 2n^3 - 5n^2 - n$	Students seem to be confused over term and sum. The question states that "first four terms of a sequence of numbers are -4, -2, 12 and 38". This should be interpreted as $u_1 = -4$, $u_2 = -2$, $u_3 = 12$ and $u_4 = 38$. However, many students interpreted it as $S_1 = -4$, $S_2 = -2$, $S_3 = 12$ and $S_4 = 38$. Upon solving, they actually got the answer

			for **part (c)** instead.
Find the nth term using the sum to n terms	**(c)**	$u_n = S_n - S_{n-1}$ $= 2n^3 - 5n^2 - n - \left[2(n-1)^3 - 5(n-1)^2 - (n-1)\right]$ $= 2n^3 - 5n^2 - n - \left[2(n^3 - 3n^2 + 3n - 1) - 5(n^2 - 2n + 1) - n + 1\right]$ $= 2n^3 - 5n^2 - n - \left(2n^3 - 11n^2 + 15n - 6\right)$ $= 6n^2 - 16n + 6$	A handful of students could not recall the general relationship between term and sum. Many students got the full credit by assuming that u_n follows a quadratic equation and proceeded to form 3 equations with 3 unknowns and solve it using G.C. However, these students must be aware that if this is not a show question, they may not know that u_n follows a quadratic equation.
Find a simplified expression for a sum using method of differences	**(d)**	$\sum_{n=10}^{2m}(u_n - u_{n-1})$ $= u_{10} - u_9$ $+ u_{11} - u_{10}$ $+ u_{12} - u_{11}$ $+ \ldots$ $+ u_{2m-1} - u_{2m-2}$ $+ u_{2m} - u_{2m-1}$ $= u_{2m} - u_9$ $= 6(2m)^2 - 16(2m) + 6 - \left[6(9)^2 - 16(9) + 6\right]$ $= 24m^2 - 32m - 342$	Many students were awarded full credit easily for this part if they had applied method of difference and replacing u_{2m} and u_9 with the expression in **part (c)** and simplifying. Some students went on to apply $\sum_{n=10}^{2m}(u_n - u_{n-1}) = \sum_{n=1}^{2m}(u_n - u_{n-1}) - \sum_{n=1}^{9}(u_n - u_{n-1})$ and often wrote the upper limits wrongly. Other students wrote $\sum_{n=10}^{2m}(u_n - u_{n-1}) = \sum_{n=10}^{2m} u_n - \sum_{n=10}^{2m} u_{n-1}$. Most went on to simplify unsuccessfully because the number of terms should be $2m - 10 + 1 = 2m - 9$.

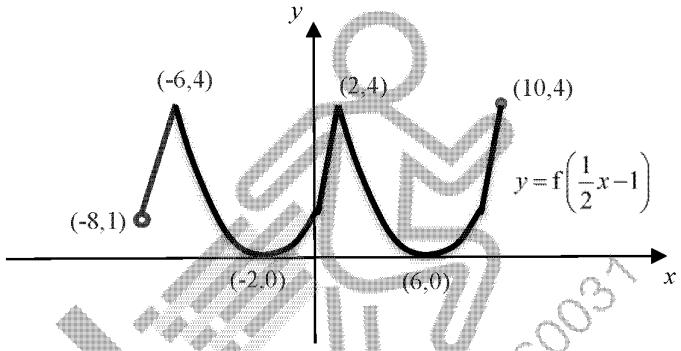
			A number of students attempted to simplify $u_n - u_{n-1}$ $= \left(6n^2 - 16n + 6\right) - \left[6\left(n-1\right)^2 - 16\left(n-1\right) + 6\right]$ Unfortunately, they were unable to handle the simplification process. The correct answer should be $12n - 22$.

Q6 Graphing Techniques [NCK]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Understand the standard form of the rational function	**(a)**	By observation, $d = 1$ Hence, $y = x + 2 + \frac{n}{x-1} = \frac{(x+2)(x-1)+n}{x-1} = \frac{x^2+x-2+n}{x-1}$ By comparison, $a = b = 1$	Students are careless in doing long-division for the function. Quite a number of students just let $y = \frac{ax^2+bx+c}{x-1} = x+2$ to find the unknowns a and b, this is incorrect as the asymptote, $y = x+2$, can never meet the curve C.
Establish condition for stationary points to exist by use of discriminant	**(b)**	$y = \frac{x^2+x+c}{x-1}$ $\frac{dy}{dx} = \frac{(2x+1)(x-1)-(x^2+x+c)}{(x-1)^2} = \frac{x^2-2x-1-c}{(x-1)^2}$ For stationary points to occur, $\frac{dy}{dx} = 0$ $x^2-2x-1-c = 0$ Hence, equation must yield 2 real roots, i.e $D > 0$ $(-2)^2 - 4(1)(-c-1) > 0$ $4 + 4c + 4 > 0$ $c > -2$	Most students can differentiate function y correctly while some are careless in applying the Quotient Rule. However, quite a number of students got stuck at $x^2-2x-1 = c$ after they let $\frac{dy}{dx} = 0$, while some students even wrote $c = \frac{2 \pm \sqrt{4-4(1)(-1)}}{2}$. Students need to read the question carefully, the question in fact asked for "range of values of c …".

Sketch graph of rational function. Recognize that there are two stationary points	**(c)**		Generally well done for the sketch of this graph. However, quite a number of students missed out the y-intercept of the graph. Students must remember to include all important/general features of the graph that they sketch. The curves should tend towards the asymptotes and not deviate away from the asymptotes towards the end.
Knowledge of graphical solutions of equations to make relevant deductions.	**(d)**	Maximum number of solutions = 2 For 2 solutions to occur, $k > 8$	Badly done. Not many students got this part of the solutions correct. A lot of students wrote maximum number of solutions as "4". They did not realized that the ellipse $\frac{(x-5)^2}{1^2}+\frac{(y-3)^2}{k^2}=1$ with centre (5, 3) will not cut the left side of the curve whereby $x<1$. For the 2 solutions to occur, they did not realize that $k>11-3 \Rightarrow k>8$.

Q7 Functions [NCK]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Sketch piece-wise graph and using the correct horizontal-line test	**(a)(i)**	 The line $y = 2$ cuts the graph of $y = \text{f}(x)$ twice. Hence f is not a one-one function and so f^{-1} does not exist.	Most students did not revise the topic thoroughly. For the sketching of graphs in Functions, it is important to reflect clearly the Domain of a function i.e. to indicate start and end points; and using ◦ to indicate that the point is not included, or • to indicate that the point is included. The given function is a piecewise function, and the difference in gradients of the 2 equations is not clearly shown. For the explanation of the existence of the inverse function, many students stated "any horizontal line, $y = k, k \in \mathbb{R}$ will cut the curve at two points." This is not true. A horizontal line will only cut the function twice when $0 < y < 4$. Hence the best approach is to give an example to illustrate the cutting of two points.
Apply condition for inverse function to exist and defining it in context of piece-wise function	**(a)(ii)**	$k = 2$ $y = (x-2)^2$ $x = 2 - \sqrt{y}$ or $2 + \sqrt{y}$ (reject as $0 < x \leq 2$) $\therefore\ \text{f}^{-1}(x) = 2 - \sqrt{x}$, $0 \leq x < 4$	Most students were able to state the value of k correctly. Many students did not consider both positive and negative when taking square root, $\pm\sqrt{}$

Apply condition for composite to exist and defining it in context of piecewise function	**(a)(iii)**	For composite fg to exist, $R_g \subseteq D_f$ From graph/observation, $R_g = (3,4]$. Hence, only second 'piece' of f is relevant: $3x-8,\ 3 < x \leq 4$ Therefore, $fg : x \mapsto 3(e^x+3)-8,\ x \in \mathbb{R},\ x \leq 0$	Most students included both equations when finding the composite function fg. They did not check the conditions for the existence of a composite function $R_g \subseteq D_f$ Only one of the two equations satisfies the condition for fg to exist. Most student who were able to find the correct composite function read the question carefully and managed to present their answers "in similar form"
Know how to handle periodic functions	**(b)(i)**	$f(25) = f(1+6\times 4) = f(1) = 1$ $f(-8) = f(4-3\times 4) = f(4) = 4$	Most students were able to evaluate calculate the outputs correctly. There were some students who left their answers as $f(25) = f(1)$ and $f(-8) = f(4)$ without evaluating the value.

Sketching and transforming periodic functions	**(b)(ii)**		This part was badly done. Most were errors in transformation due to a lack of understand of periodic functions. Many students did not realize that they could reuse the graph in (i). There were some students who obtained the shape but did not indicate the end points correctly (◦ to indicate that the point is not included, • to indicate included)

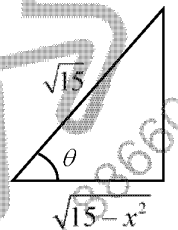
Q8 Parametric Equations [CSL]

Assessment Objectives	Solution	Examiner's Feedback
Apply integration by substitution	**(a)**	Some students used $\frac{\mathrm{d}\theta}{\mathrm{d}x}$ instead of $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ Many students got stuck after they reached $=15\int\cos^2\theta\,\mathrm{d}\theta$ Students need to practice their integration more, especially the use of formulas from the MF26. Some students did not know how to convert $\cos\theta$ back to x

$$\int\sqrt{15-x^2}\,\mathrm{d}x$$

$$x=\sqrt{15}\sin\theta$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sqrt{15}\cos\theta$$



$$=\int\sqrt{15-x^2}\,\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$$

$$=\int\sqrt{15-15\sin^2\theta}\sqrt{15}\cos\theta\,\mathrm{d}\theta$$

$$=\int\sqrt{15\left(1-\sin^2\theta\right)}\sqrt{15}\cos\theta\,\mathrm{d}\theta$$

$$=15\int\cos^2\theta\,\mathrm{d}\theta$$

$$=15\int\frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta$$

$$=\frac{15}{2}\left(\frac{\sin 2\theta}{2}+\theta\right)+C$$

$$=\frac{15}{2}\left(\sin\theta\cos\theta+\theta\right)+C$$

$$=\frac{15}{2}\left[\frac{x}{\sqrt{15}}\cdot\frac{\sqrt{15-x^2}}{\sqrt{15}}+\sin^{-1}\left(\frac{x}{\sqrt{15}}\right)\right]+C$$

$$=\frac{1}{2}x\sqrt{15-x^2}+\frac{15}{2}\sin^{-1}\left(\frac{x}{\sqrt{15}}\right)+C$$

Solve point of intersection for parametric and cartesian equations	**(b)**	**Method ①:** $x = 6\cos\theta, \quad y = 2\sqrt{2}\sin\theta \quad$ — ① $x^2 + y^2 = 15 \quad$ — ② $(6\cos\theta)^2 + \left(2\sqrt{2}\sin\theta\right)^2 = 15$ $36\cos^2\theta + 8\sin^2\theta = 15$ $36(1 - \sin^2\theta) + 8\sin^2\theta = 15$ $36 - 36\sin^2\theta + 8\sin^2\theta = 15$ $28\sin^2\theta = 21$ $\sin^2\theta = \frac{3}{4}$ $\sin\theta = \frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$ (rej. $\because \sin\theta > 0$ for P) $\theta = \frac{\pi}{3}$ When $\theta = \frac{\pi}{3}, \quad x = 6\cos\frac{\pi}{3} = 6\left(\frac{1}{2}\right) = 3$ $y = 2\sqrt{2}\sin\frac{\pi}{3} = 2\sqrt{2}\left(\frac{\sqrt{3}}{2}\right) = \sqrt{6}$ $\therefore P\left(3, \sqrt{6}\right)$ **Method ②:** $x = 6\cos\theta, \quad y = 2\sqrt{2}\sin\theta$ Using $\cos^2\theta + \sin^2\theta = 1$, $\left(\frac{x}{6}\right)^2 + \left(\frac{y}{2\sqrt{2}}\right)^2 = 1$	Many students failed to take $\pm$ when they square root $\sin^2\theta = \frac{3}{4}$ Some students did not give reason for rejection of negative root.

$$\frac{x^2}{36}+\frac{y^2}{8}=1 \quad \text{—} \quad ①$$

$x^2+y^2=15$

$y^2=15-x^2 \quad \text{—} \quad ②$

Substitute ② into ①:

$$\frac{x^2}{36}+\frac{15-x^2}{8}=1$$

$8x^2+36\left(15-x^2\right)=288$

$28x^2=252$

$x^2=9$

$x=3$ or -3 $\left(\text{rej. } \because x>0 \text{ for } P\right)$

When $x=3$, $y^2=15-9=6$

$y=\sqrt{6}$ or $-\sqrt{6}$ $\left(\text{rej. } \because y>0 \text{ for } P\right)$

$\therefore P\left(3, \sqrt{6}\right)$

Area bounded by curves	**(c)**	**Method ①:** Area of region $R = \underbrace{\int_0^3 y_2 \, dx}_{\text{curve } C_2} - \underbrace{\int_0^3 y_1 \, dx}_{\text{curve } C_1}$ **For C_2:** $\int_0^3 y_2 \, dx = \int_0^3 \sqrt{15 - x^2} \, dx$ $= \left[\frac{1}{2} x\sqrt{15 - x^2} + \frac{15}{2} \sin^{-1}\left(\frac{x}{\sqrt{15}} \right) \right]_0^3$ $= \frac{3}{2}\sqrt{6} + \frac{15}{2} \sin^{-1}\left(\frac{3}{\sqrt{15}} \right)$ $= \frac{3}{2}\sqrt{6} + \frac{15}{2} \sin^{-1}\left(\frac{\sqrt{15}}{5} \right)$	Some students did not realise they could use the earlier result for C_2. Some students found $C_2 - C_1$

For C_1:

$$\int_0^3 y_1 \,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 2\sqrt{2}\sin\theta \frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 2\sqrt{2}\sin\theta(-6\sin\theta)\mathrm{d}\theta$$

$$= -6\sqrt{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 2\sin^2\theta \,\mathrm{d}\theta$$

$$= -6\sqrt{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} (1-\cos 2\theta)\mathrm{d}\theta$$

$$= -6\sqrt{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{\frac{\pi}{3}}$$

$$= -6\sqrt{2}\left[\left(\frac{\pi}{3} - \frac{\sin\left(\frac{2\pi}{3}\right)}{2}\right) - \left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right)\right]$$

$$= -6\sqrt{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} - \frac{\pi}{2}\right)$$

$$= -6\sqrt{2}\left(-\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \sqrt{2}\pi + \frac{3\sqrt{6}}{2}$$

Area of region $R = \frac{3}{2}\sqrt{6} + \frac{15}{2}\sin^{-1}\left(\frac{\sqrt{15}}{5}\right) - \sqrt{2}\pi - \frac{3}{2}\sqrt{6}$

$$= \frac{15}{2}\sin^{-1}\left(\frac{\sqrt{15}}{5}\right) - \sqrt{2}\pi$$

Some students put in the limits the wrong way. It is not correct to assume that the smaller value is always the lower limit.

Students need to note the correct form that the question is asking for.

ie. $\frac{3}{\sqrt{15}}$ is not acceptable.

where $m=\frac{15}{2}$ and $k=\frac{1}{5}$

Method ②: [Not possible]

Area of region $R=\underbrace{\int_0^3 y_2\,\mathrm{d}x}_{\text{curve } C_2}-\underbrace{\int_0^3 y_1\,\mathrm{d}x}_{\text{curve } C_1}$

For C_2:

$$\int_0^3 y_2\,\mathrm{d}x=\int_0^3\sqrt{15-x^2}\,\mathrm{d}x$$

$$=\left[\frac{1}{2}x\sqrt{15-x^2}+\frac{15}{2}\sin^{-1}\left(\frac{x}{\sqrt{15}}\right)\right]_0^3$$

$$=\frac{3}{2}\sqrt{6}+\frac{15}{2}\sin^{-1}\left(\frac{3}{\sqrt{15}}\right)$$

$$=\frac{3}{2}\sqrt{6}+\frac{15}{2}\sin^{-1}\left(\frac{\sqrt{15}}{5}\right)$$

For C_1:

$$\frac{y^2}{8}=1-\frac{x^2}{36}$$

$$y^2=8-\frac{2}{9}x^2$$

$$y=\sqrt{8-\frac{2}{9}x^2}\ (\text{rej}-\text{ve}\because y>0)$$

$$\int_0^3 y_1\,\mathrm{d}x=\int_0^3\sqrt{8-\frac{2}{9}x^2}\,\mathrm{d}x$$

$$=\frac{\sqrt{2}}{3}\int_0^3\sqrt{36-x^2}\,\mathrm{d}x$$

$$=\ldots$$

hd

Students who tried method 2 can at most get the marks for obtaining **C_2**.

The expression for **C_1** is not integrable unless a substitution is given.

Q9 Applications of Differentiation [KP]

Assessment Objectives	Solution		Examiner's Feedback
Formulate an equation based on given conditions in the context of the question.	**(a)**	Length of $BC = 2r\cos\theta$ Length of $CD = 2r\sin\theta$ $P = 2[AB + BC + CD]$ $= 2[4r + 2r\cos\theta + 2r\sin\theta]$ $= 4r(2 + \cos\theta + \sin\theta)$ (shown)	This part of the question was generally well done. Majority of the students were able to find the length of *AF*/*BC* and *CD*/*EF* using the properties of a right-angled triangle, and eventually show the perimeter of *ABCDEF*.
Solve local maxima and minima problems, and use the first derivative test or the second derivative test.	**(b)**	$\frac{dP}{d\theta} = 4r(-\sin\theta + \cos\theta)$ At maximum P, $4r(-\sin\theta + \cos\theta) = 0$ $\sin\theta = \cos\theta$ $\tan\theta = 1$ $\theta = \frac{\pi}{4}$ **Method ①:** θ | $\left(\frac{\pi}{4}\right)^-$ | $\frac{\pi}{4}$ | $\left(\frac{\pi}{4}\right)^+$ $\frac{dP}{d\theta}$ | >0 | 0 | <0 | ／ | — | ＼ Hence, P is maximum when $\theta = \frac{\pi}{4}$.	This part of the question was generally well-attempted. Students who attempted this part of the question generally knew the general procedure to answer the question. However, a significant number of students overlooked some of the steps and/or made algebraic slips. Common mistakes include: 1. Not knowing r is a constant and treated it like a variable. 2. Differentiating with respect to r instead of θ. 3. $\frac{dP}{d\theta} = (-\sin\theta + \cos\theta)$ [Missing $4r$] 4. $\frac{dP}{d\theta} = 4r(2 - \sin\theta + \cos\theta)$ 5. $\sin\theta = \cos\theta \Rightarrow \tan\theta = 0$

Method ②:

$$\frac{\mathrm{d}^2P}{\mathrm{d}\theta^2} = 4r\left(-\cos\theta - \sin\theta\right)$$

When $\theta = \frac{\pi}{4}$,

$$\frac{\mathrm{d}^2P}{\mathrm{d}\theta^2} = 4r\left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right) < 0$$

Hence, P is maximum when $\theta = \frac{\pi}{4}$.

$$\text{Maximum Distance} = 4r\left(2 + \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)$$

$$= 4r\left(2 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$= 4r\left(2 + \sqrt{2}\right) \text{ metres}$$

6. $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$
7. Omitting derivative test.
8. Using values such as $\frac{\pi}{4} + 0.1$ for 1st derivative test instead of $\left(\frac{\pi}{4}\right)^-$, $\frac{\pi}{4}$ and $\left(\frac{\pi}{4}\right)^+$.
9. Not stating the conclusion of the derivative test.
10. Poor algebraic manipulation, especially dealing with square roots, i.e. rationalizing the denominator.

Students should take note of the following:

1. Learn to identify which is the variable and which is a constant. Not all letters denote variables.
2. Practise the techniques of differentiation (and integration). This is important when answering calculus questions.
3. Get exposed to solving trigo equations. Use the calculator to check if

			needed. 4. Always remember to perform the derivative test (1^{st} or 2^{nd}) and state the conclusion, unless the question says there is no need for the derivative test. 5. Review the basic algebraic manipulation skills, e.g. rationalizing denominator, completing the square, partial fractions, long division etc.

Solve for unknown fixed constants when given conditions in the context of the question.	**(c)**	3 minutes = 180 seconds Time to complete one loop $= \dfrac{4r\left(2+\sqrt{2}\right)}{6}$ To find maximum value of r, $\dfrac{4r\left(2+\sqrt{2}\right)}{6} = 180$ $r = \dfrac{180\times 6}{4\left(2+\sqrt{2}\right)}$ $= \dfrac{270}{\left(2+\sqrt{2}\right)} \times \dfrac{2-\sqrt{2}}{2-\sqrt{2}}$ $= 135\left(2-\sqrt{2}\right)$ $= 270 - 135\sqrt{2}$	This part of the question was poorly attempted. Weak algebraic manipulation was observed. Common mistakes include: 1. Mis-interpreting the question and used rate of change to attempt this question. 2. Poor algebraic manipulation, especially dealing with square roots, i.e. rationalizing the denominator. Students should take note of the following: 1. Review the basic algebraic manipulation skills, e.g. rationalizing denominator, completing the square, partial fractions, long division etc.
Reason and justify the affordability/feasibility in the context of the question.	**(d)**	Area of $\Delta BCD = \dfrac{1}{2}(2r)(2r\cos\theta\sin\theta)$ $= 2r^2\cos\theta\sin\theta$ Area of $ABCDEF = (4r)(2r) + 2\left(2r^2\cos\theta\sin\theta\right)$ $= 8r^2 + 4r^2\cos\theta\sin\theta$ When $\theta = \dfrac{\pi}{4}$ and $r = 270 - 135\sqrt{2}$,	This part of the question was poorly attempted, although students knew to find the cost needed and compare with \$10000. Common mistakes include: 1. Unable to formulate the area of ΔBCD. 2. Missing out the square, i.e

		Cost of planting grass for *ABCDEF* $= 0.15 \times \left[8\left(270 - 135\sqrt{2}\right)^2 + 4\left(270 - 135\sqrt{2}\right)^2 \cos\frac{\pi}{4}\sin\frac{\pi}{4} \right]$ $= \$9380.75 < \$10000$ Hence, management can afford to cover the entire shape *ABCDEF* with grass.	$(4r)(2r) = 8r$ and $2\left(2r^2 \cos\theta \sin\theta\right)$ $= 4r\cos\theta\sin\theta$

Q10 Sequences and Series [LWY]

Assessment Objectives	Solution		Examiner's Feedback
Identify that the sequence is a G.P.	**(a)**	nth day | Number of daily views at the end of nth day 1 | 1196 2 | $3(1196)$ 3 | $3^2(1196)$ G.P. with first term = 1196 and common ratio = 3 Number of daily views at the end of the third day $=3^2(1196)$ $=10764$	This part was generally well done. Students were able to recognize this was a GP and identified the common ratio correctly. However, some students did not read the question carefully. The question asked for the **daily** views (not the total views). Hence the formula for the general tern, $u_n = ar^{n-1}$ should be used.
Apply the formula for the general term and make a decision based on the context of the question	**(b)**	Total number of views at the end of the 7th day $=\frac{1196(3^7-1)}{3-1}$ $=1307228$ $<5000\ 000$ The video will not go viral.	This part was generally well done. Some students did not read the question carefully. The question asked for the **total** number of views. Hence the formulae for the

			sum, $S_n = \frac{a(r^n - 1)}{r - 1}$ should be used. There were some students who found the sum without the formula (doable in this question since the number of terms was small) – they are strongly encouraged to recall and apply the GP sum. Some students did not explain clearly why the video was not viral. There is a need to compare the value found to the criterion condition in order to make the conclusion.
Identify that the sequence is a A.P. and apply the sum to first n terms	**(c)**	$\frac{n}{2}[2(576)+(n-1)(780)] > 100\,000$ $576n + 390n^2 - 390n > 100\,000$ $390n^2 + 186n - 100\,000 > 0$ Using G.C., $n < -16.253$ or $n > 15.776$	This part was not well done. Students were able to recognize this was an AP and identified the common difference. A few students mixed up the first

		OR n | $390n^2+186n-100\,000$ | 15 | -9460 | <0 16 | 2816 | >0 17 | 15872 | >0 Least $n=16$	term and the common difference. A significant percentage of students did not read the question carefully. The question asked for the **total** number of comments. Hence the formulae for the sum, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ should be used. A few students who tried to do it by trial and error – this would take too long if n is a large number. Students are strongly encouraged to show their workings in arriving at their final answer. If the final answer is wrong and there is no working for solving the quadratic inequality, 2 marks would be lost.

<table>
<tr>
<td>Derive a pattern and apply the G.P. sum of first n terms</td>
<td>(d)</td>
<td>
Total number of comments at the end of Day 16
$=\frac{16}{2}\left[2(576)+(16-1)(780)\right]$ OR $=100\,000+2816$ (from G.C. table)
$=102816$ $=102816$
<table>
<tr><th>n</th><th>Start of Day</th><th>End of Day</th></tr>
<tr><td>1</td><td>$102\,816-w$</td><td>$1.03(102\,816-w)$
$=1.03(102\,816)-1.03w$</td></tr>
<tr><td>2</td><td>$1.03(102\,816)-1.03w-w$</td><td>$1.03\left[1.03(102\,816)-1.03w-w\right]$
$=(1.03)^2(102\,816)-(1.03)^2w-1.03w$</td></tr>
<tr><td>3</td><td>$(1.03)^2(102\,816)-(1.03)^2w-1.03w-w$</td><td>$(1.03)^3(102\,816)-(1.03)^3w-(1.03)^2w-1.03w$</td></tr>
</table>
Number of comments by the end of Day n
$=(1.03)^n(102\,816)-(1.03)^nw-(1.03)^{n-1}w-\ldots-1.03w$
$=(1.03)^n(102\,816)-\left[(1.03)^nw+(1.03)^{n-1}w+\ldots+1.03w\right]$
$=(1.03)^n(102\,816)-\frac{1.03w\left[(1.03)^n-1\right]}{1.03-1}$
$=(1.03)^n(102\,816)-\frac{103w}{3}\left[(1.03)^n-1\right]$ (shown)
where $M=102816$
</td>
<td>
This part was not well done.

A significant percentage of students failed to write the finite series clearly before applying the formula for GP sum. A minimum of the first 2 terms must be written so that a GP can be identified and the last term must be written to indicate the sum is finite.

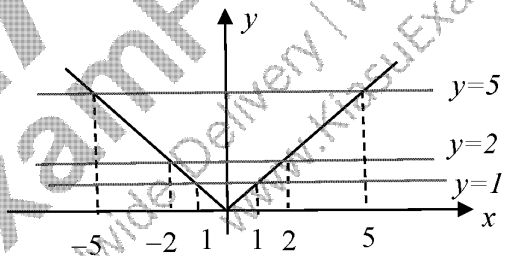
Students are strongly encouraged to write the finite series for Day 3 or more if they are unsure of the pattern.

Most students failed to identify the total number of comments at the start of Day 1 which was the total number of comments at the end of Day 16 (from part (c)).
</td>
</tr>
</table>

Use the previous result to find a range for the unknown	**(e)**	$(1.03)^{31}(102\,816)-\frac{103w}{3}\left[(1.03)^{31}-1\right]\le 0$ $\frac{103w}{3}\left[(1.03)^{31}-1\right]\ge(1.03)^{31}(102\,816)$ $w\ge 4990.961031$ $w\ge 4991$	This part was poorly attempted. Students should attempt this part by using the result in part (d) even though they might not be able to show the result. Some students failed to realize the answer should be a **range** of values of w.

Q1 Inequalities [NCK]

Assessment Objectives	Solution	Examiner's Feedback
Standard procedure in using algebraic method in solving of inequality	$\frac{x^2+2x-5}{x^2-2x} < 2$ $\frac{x^2+2x-5}{x^2-2x} - 2 < 0$ $\frac{x^2+2x-5-2x^2+4x}{x^2-2x} < 0$ $\frac{-x^2+6x-5}{x(x-2)} < 0$ $\frac{x^2-6x+5}{x(x-2)} > 0$ $\frac{(x-1)(x-5)}{x(x-2)} > 0$  $\therefore\ x < 0$ or $1 < x < 2$ or $x > 5$.	This part of the question was poorly attempted. Many students were unable to solve the inequality completely using the algebraic method. Common mistakes include: 1. Making algebraic slips when attempting to combine two terms into a single fraction. 2. Use a graphical method when question stated not to use a calculator. 3. Did not change the inequality sign, i.e. $\frac{-x^2+6x-5}{x(x-2)} < 0$ $\Rightarrow \frac{x^2-6x+5}{x(x-2)} < 0$ 4. Including the equality sign for critical values of 1 and 5, i.e. $x < 0$ or $1 \le x < 2$ or $x \ge 5$ 5. Cross multiplying the inequality, i.e. $x^2+2x-5 < 2\left(x^2-2x\right)$

		Students should take note of the following: 1. Learn/Revise the standard procedure for solving an inequality algebraically. 2. Read the question carefully to determine if a calculator can be used. 3. Pay attention to the inequality sign, especially when a normal calculator was used to determine the roots/factors. 4. Never cross multiply when solving inequality, unless the term is always positive or always negative. 5. Use a GC to check the solution of the inequality obtained.
Use of result to solve a new inequality by establishing a link(and hence replacement) between both inequalities. Solving of inequalities involving modulus. [HOT]	Solution of $\frac{x^2+2x-5}{x^2-2x}>2$ is the complementary of solution of $\frac{x^2+2x-5}{x^2-2x}<2$ So, replacing x with $\lvert x\rvert$, solution of $\frac{x^2+2\lvert x\rvert-5}{x^2-2\lvert x\rvert}>2$ will be $0<\lvert x\rvert<1$ or $2<\lvert x\rvert<5$  	This part of the question was poorly attempted. Common mistakes include: 1. Unable to observe the necessary replacement of x, either omitting totally or used an incorrect replacement. 2. Solve the inequality from scratch instead of using "Hence" as stated in the question. 3. Failing to identify that the complementary of the

	For $0 < \|x\| < 1$, $-1 < x < 1,\ x \neq 0$ For $2 < \|x\| < 5$, $-5 < x < -2$ or $2 < x < 5$ Thus, range of values: $-5 < x < -2$ or $2 < x < 5$ or $-1 < x < 1,\ x \neq 0$	solution is needed. 4. Unable to solve the inequality in x, but gave the inequality in $\|x\|$ instead. 5. Did not exclude $x = 0$ for the solution. Students should take note of the following: 1. Learn/Revise how to solve an inequality involving modulus. A graph would certainly help in the process. 2. Pay attention to the inequality sign to determine if the complementary of the solution is needed.

Q2 Maclaurin Series [CSL]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Apply product rule	**(a)**	$y = \mathrm{e}^{-x}\sin x + x - 1$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-x}\cos x - \mathrm{e}^{-x}\sin x + 1$ $= \mathrm{e}^{-x}(\cos x - \sin x) + 1$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{-x}(-\sin x - \cos x) - \mathrm{e}^{-x}(\cos x - \sin x)$ $= -2\mathrm{e}^{-x}\cos x$ where $k = -2$ (shown)	This part of the question was well attempted. Most students were able to obtain full credit for this part of the question. Common mistakes include: 1. Making algebraic slips when differentiating, i.e. forgot the negative sign. 2. $\frac{\mathrm{d}y}{\mathrm{d}x} = (-\mathrm{e}^{-x})\cos x + 1$ [Did not use product rule but differentiating the product of the terms separately] Students should take note of the following: 1. Practise the techniques of differentiation (and integration). This is important when answering calculus questions. 2. Get exposed to solving trigo equations. Use the calculator to check if needed.
Carry out repeated differentiation	**(b)**	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -2\mathrm{e}^{-x}(-\sin x) - (-2\mathrm{e}^{-x})\cos x$ $= 2\mathrm{e}^{-x}(\sin x + \cos x)$	This part of the question was well attempted. Most students were able to obtain full credit for this part of the question. Common mistakes include:

		When $x=0: \mathrm{f}(0)=-1$ $\mathrm{f}'(0)=2$ $\mathrm{f}''(0)=-2$ $\mathrm{f}'''(0)=2$ $y=-1+2x-2\cdot\frac{x^2}{2!}+2\cdot\frac{x^3}{3!}+\ldots$ $=-1+2x-x^2+\frac{x^3}{3}+\ldots$	1. $2\cdot\frac{x^3}{3!}=\frac{2}{3}x^3$ 2. Evaluating the values of $\mathrm{f}(0)$, $\mathrm{f}'(0)$, $\mathrm{f}''(0)$ and $\mathrm{f}'''(0)$ incorrectly. Students should take note of the following: 1. Learn/Revise the procedure of finding the Maclaurin Series expansion.
Apply small angle approximation	**(c)**	$\frac{\mathrm{e}^{-x}\sin x+x-1}{\cos 2x}=\frac{-1+2x-x^2+\frac{x^3}{3}+\ldots}{1-\frac{(2x)^2}{2!}+\ldots}$ $=\left(-1+2x-x^2+\frac{x^3}{3}+\ldots\right)\left(1-2x^2\right)^{-1}$ $=\left(-1+2x-x^2+\frac{x^3}{3}+\ldots\right)\left[1+(-1)\left(-2x^2\right)+\ldots\right]$ $=\left(-1+2x-x^2+\frac{x^3}{3}+\ldots\right)\left(1+2x^2+\ldots\right)$ $=-1-2x^2+2x+4x^3-x^2+\frac{x^3}{3}+\ldots$ $=-1+2x-3x^2+\frac{13}{3}x^3+\ldots$	This part of the question was poorly attempted. Common mistakes include: 1. $\cos 2x=1-\frac{x^2}{2!}$ [Fail to replace x with $2x$] 2. Unable to apply Binomial expansion to $\left(1-2x^2\right)^{-1}$. 3. $\left(1-2x^2\right)^{-1}=1-\frac{1}{2x^2}$ [Incorrect expansion] 4. Performing long division Students should take note of the following: 1. Learn/Revise how to apply Binomial Expansion 2. Review the strategy to express an algebraic fraction as a power series

			using the Binomial Series expansion.

Q3 Differential Equations [CSL]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Apply partial fractions Solve differential equation to obtain a general solution followed by substituting condition to obtain a particular solution	**(a)**	**Method 1** $\frac{1}{P(13-2P)} = \frac{M}{P} + \frac{N}{13-2P}$ $1 = M(13-2P) + NP$ When $P = 0$, $M = \frac{1}{13}$ When $P = \frac{13}{2}$, $N = \frac{2}{13}$ $\frac{1}{P(13-2P)} = \frac{1}{13P} + \frac{2}{13(13-2P)}$ $= \frac{1}{13}\left(\frac{1}{P} + \frac{2}{13-2P}\right)$ $\frac{dP}{dt} = \frac{1}{26}P(13-2P)$ $\int \frac{1}{P(13-2P)}\,dP = \frac{1}{26}\int 1\,dt$ $\frac{1}{13}\int \frac{1}{P} + \frac{2}{(13-2P)}\,dP = \frac{1}{26}\int 1\,dt$ $\int \frac{1}{P}\,dP - \int \frac{-2}{(13-2P)}\,dP = \frac{1}{2}\int 1\,dt$ $\ln\lvert P\rvert - \ln\lvert 13-2P\rvert = \frac{1}{2}t + C$ $\ln\left\lvert\frac{P}{13-2P}\right\rvert = \frac{1}{2}t + C$ $\left\lvert\frac{P}{13-2P}\right\rvert = e^{\frac{1}{2}t+C}$	Most students were able to observe that this is a variable separable differential equation. Students had two main methods, Method 1 and 2. Many students did not put modulus sign for $\ln\lvert 13-2P\rvert$

$$\frac{P}{13-2P} = \pm e^{\frac{1}{2}t+C}$$

$$\frac{P}{13-2P} = \pm e^{\frac{1}{2}t} \cdot e^{C}$$

$$\frac{P}{13-2P} = Ae^{\frac{1}{2}t}$$

$$P = 13Ae^{\frac{1}{2}t} - 2APe^{\frac{1}{2}t}$$

$$P\left(1+2Ae^{\frac{1}{2}t}\right) = 13Ae^{\frac{1}{2}t}$$

$$P = \frac{13Ae^{\frac{1}{2}t}}{1+2Ae^{\frac{1}{2}t}}$$

$$P = \frac{13A}{e^{-\frac{1}{2}t}+2A}$$

When $t = 0$, $P = 2$,

$$2 = \frac{13A}{1+2A}$$

$$2+4A = 13A$$

$$9A = 2$$

$$A = \frac{2}{9}$$

$$P = \frac{13\left(\frac{2}{9}\right)}{e^{-\frac{1}{2}t} + 2\left(\frac{2}{9}\right)}$$

$$= \frac{26}{9e^{-\frac{1}{2}t} + 4} \text{ (shown)}$$

Method 2

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{26}P(13-2P)$$

$$\int \frac{1}{P(13-2P)}\,\mathrm{d}P = \frac{1}{26}\int 1\,\mathrm{d}t$$

$$\int \frac{1}{13P-2P^2}\,\mathrm{d}P = \frac{1}{26}\int 1\,\mathrm{d}t$$

$$-\frac{1}{2}\int \frac{1}{\left(P-\frac{13}{4}\right)^2-\left(\frac{13}{4}\right)^2}\,\mathrm{d}P = \frac{1}{26}\int 1\,\mathrm{d}t$$

$$-\frac{1}{2}\frac{1}{2\left(\frac{13}{4}\right)}\ln\left|\frac{P-\frac{13}{4}-\frac{13}{4}}{P-\frac{13}{4}+\frac{13}{4}}\right| = \frac{1}{26}t + C$$

$$\ln\left|\frac{P-\frac{13}{2}}{P}\right| = -\frac{1}{2}t + C'$$

$$\left|\frac{P-\frac{13}{2}}{P}\right| = \mathrm{e}^{-\frac{1}{2}t+C'}$$

$$\frac{P-\frac{13}{2}}{P}=\pm e^{-\frac{1}{2}t+C'}$$

$$\frac{P-\frac{13}{2}}{P}=\pm e^{-\frac{1}{2}t}\cdot e^{C'}$$

$$\frac{P-\frac{13}{2}}{P}=Ae^{-\frac{1}{2}t}$$

$$P=\frac{13}{2}+APe^{-\frac{1}{2}t}$$

$$P\left(1-Ae^{-\frac{1}{2}t}\right)=\frac{13}{2}$$

$$P=\frac{13}{2-4Ae^{-\frac{1}{2}t}}$$

When $t=0$, $P=2$,

$$2=\frac{13}{2-4A}$$

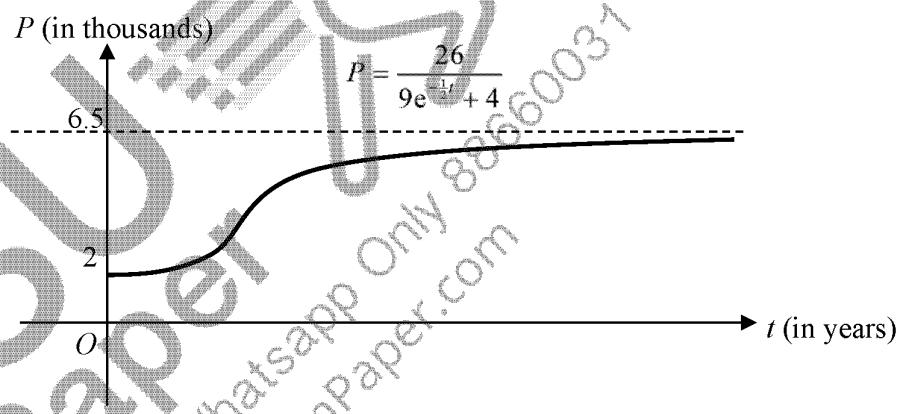
$$4-8A=13$$

$$A=-\frac{9}{8}$$

$$P=\frac{13}{2-4\left(-\frac{9}{8}\right)e^{-\frac{1}{2}t}}$$

$$=\frac{26}{9e^{-\frac{1}{2}t}+4}\ (\text{shown})$$

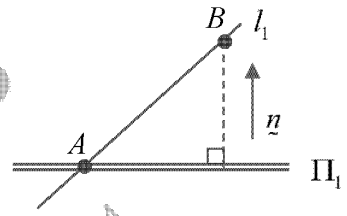
Understand the context and solve	**(b)**	When $P = 4$, $$4 = \frac{26}{9e^{-\frac{1}{2}t} + 4}$$ $$4\left(9e^{-\frac{1}{2}t} + 4\right) = 26$$ $$9e^{-\frac{1}{2}t} = \frac{5}{2}$$ $$e^{-\frac{1}{2}t} = \frac{5}{18}$$ $$-\frac{1}{2}t = \ln\left(\frac{5}{18}\right)$$ $$t = -2\ln\left(\frac{5}{18}\right)$$ $$t = 2.56$$ It takes 2.56 months for the number of people who downloaded Ginseng Impact to double since the launch.	Table method cannot be used as the number of months is not an integer.

Understand that in the long run implies that time approaches infinity	**(c)**	As $t \to \infty$, $e^{-\frac{1}{2}t} \to 0$, $P \to \frac{26}{4}$ Number of people that downloaded Ginseng Impactful in the long run is $\frac{26}{4}(1000) = 6500$.	Students who substituted a large value of t to obtain the answer did not get the credit. They need to understand the concept of limits.
Sketch graph in the context of the question	**(d)**		Poor shape for most students.

Q4 Vectors [LWY]

Assessment Objectives	Solution		Examiner's Feedback
Find acute angle between a plane and line	**(a)**	Vector equation of the line l is $\underset{\sim}{r} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$ Let α be the acute angle between the normal vector of plane Π_1 and line l. $\alpha = \cos^{-1} \dfrac{\left\lvert \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \right\rvert}{\left\lVert \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\rVert \left\lVert \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \right\rVert}$ $= \cos^{-1} \left\lvert \dfrac{3}{\sqrt{6}\sqrt{14}} \right\rvert$ $= 70.893^\circ$ or 1.2373 $\therefore$ acute angle between the plane Π_1 and line l is $= 90^\circ - 70.893^\circ$ or $\dfrac{\pi}{2} - 1.2373$ $= 19.1^\circ$ or 0.333	Presentation issues abound. Some examples are: 1) In writing the line $\underset{\sim}{r} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ the $\underset{\sim}{r}$ is either missing or without the tilde. 2) $\left\lvert \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \right\rvert$ is mistakenly written as $\left\lVert \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\rVert \bullet \left\lVert \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \right\rVert$ which means an entirely different thing altogether. Candidates also have to be aware that, in the case of finding angle between line and plane, they have to subtract the result from 90^o or use $\sin^{-1}$

Find the point of intersection between a line and a plane	**(b)**	Substitute equation of line into the equation of plane: $\left[\begin{pmatrix}-2\\4\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\-1\end{pmatrix}=3$ $-3+3\lambda=3$ $\lambda=2$ When $\lambda=2$, $r=\begin{pmatrix}-2\\4\\3\end{pmatrix}+2\begin{pmatrix}3\\-2\\1\end{pmatrix}=\begin{pmatrix}4\\0\\5\end{pmatrix}$ Coordinates of the point of intersection is (4, 0, 5).	Some candidates missed out on giving the requirement of the question and that is to give the coordinates as the answer.

Find distance between a point and a plane	**(c)**	**Method ①:** Perpendicular distance from B to Π_1 $= \left\lvert \frac{\overrightarrow{AB} \cdot \underset{\sim}{n}}{\lvert \underset{\sim}{n} \rvert} \right\rvert$ $= \left\lvert \frac{\left[\begin{pmatrix} 10 \\ -4 \\ 7 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} \right] \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}}{\left\lvert \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\rvert} \right\rvert$ $= \left\lvert \frac{\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}}{\sqrt{6}} \right\rvert$ $= \left\lvert \frac{6}{\sqrt{6}} \right\rvert$ $= \sqrt{6}$ units  **Method ②:** Equation of line that is parallel to normal vector and passing through (10, -4, 7) is: $\underset{\sim}{r} = \begin{pmatrix} 10 \\ -4 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \mu \in \mathbb{R}$	Method 1 Some candidates did not use the formula correctly: 1) they used $\overrightarrow{OB}$ instead of $\overrightarrow{AB}$ 2) they missed out on $\lvert \underset{\sim}{n} \rvert$ in the denominator 3) they used $\lvert \overrightarrow{AB} \rvert$ instead of $\lvert \underset{\sim}{n} \rvert$ in the denominator

At the point of intersection,

$$\left[\begin{pmatrix}10\\-4\\7\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\-1\end{pmatrix}=3$$

$$9+6\mu=3$$

$$\mu=-1$$

When $\mu=1$,

$$\underline{r}=\begin{pmatrix}10\\-4\\7\end{pmatrix}+\begin{pmatrix}-2\\-1\\1\end{pmatrix}=\begin{pmatrix}8\\-5\\8\end{pmatrix}$$

Perpendicular distance from B to Π_1

$$=\left|\begin{pmatrix}8\\-5\\8\end{pmatrix}-\begin{pmatrix}10\\-4\\7\end{pmatrix}\right|$$

$$=\left|\begin{pmatrix}-2\\-1\\1\end{pmatrix}\right|$$

$$=\sqrt{(-2)^2+(-1)^2+1^2}$$

$$=\sqrt{6}\text{ units}$$

Method 2

Some candidates took $\left|\begin{pmatrix}8\\-5\\8\end{pmatrix}\right|$ directly as the perpendicular distance. They should be more careful

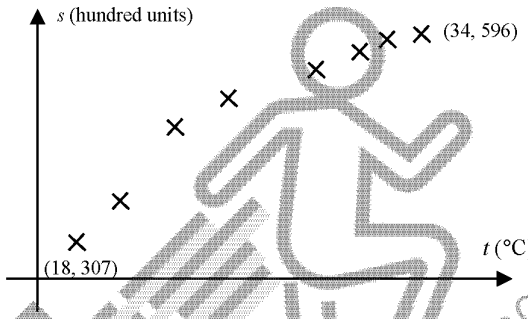
Find cartesian equation of a plane	**(d)**	A vector perpendicular to the plane Π_2 is $\begin{pmatrix}3\\-2\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}1\\5\\7\end{pmatrix}$ Equation of plane Π_2 is $\underset{\sim}{r}\cdot\begin{pmatrix}1\\5\\7\end{pmatrix}=\begin{pmatrix}-2\\4\\3\end{pmatrix}\cdot\begin{pmatrix}1\\5\\7\end{pmatrix}$ $x+5y+7z=39$	Some candidates missed out on giving the requirement of the question and that is to give the cartesian equation of the plane as the answer.
Find the equation of the line of intersection between 2 planes without GC	**(e)**	Note that point $(4, 0, 5)$ lies on both planes Π_1 and Π_2. A vector parallel to the line of intersection of both planes is $\begin{pmatrix}2\\1\\-1\end{pmatrix}\times\begin{pmatrix}1\\5\\7\end{pmatrix}=\begin{pmatrix}12\\-15\\9\end{pmatrix}=3\begin{pmatrix}4\\-5\\3\end{pmatrix}$ Vector equation of the line that lies in both planes is $\underset{\sim}{r}=\begin{pmatrix}4\\0\\5\end{pmatrix}+\mu\begin{pmatrix}4\\-5\\3\end{pmatrix},\mu\in\mathbb{R}$	This part is not well attempted. Many candidates are aware that to obtain the direction of the line, they can take the cross-product of the normals of the two planes involved. What they did not realize is that the common point to both planes is the point found on (b)

Q5 Discrete Random Variables [LL]

Assessment Objectives	Solution		Examiner's Feedback
Tables of Outcomes is not essential. Students are able to make a systematic list or other heuristics and techniques to identify all possible cases.		Tables of outcomes $\begin{array}{\|c\|c\|c\|c\|c\|}\hline & 1 & 2 & 3 & 4 \\ \hline 1 & 3 & 3 & 4 & 5 \\ \hline 2 & 3 & 6 & 5 & 6 \\ \hline 3 & 4 & 5 & 9 & 7 \\ \hline 4 & 5 & 6 & 7 & 12 \\ \hline \end{array}$ $\mathrm{P}(\text{spin}=1)=\frac{144}{360}=\frac{2}{5}=\frac{4}{10}$ $\mathrm{P}(\text{spin}=2)=\frac{108}{360}=\frac{3}{10}$ $\mathrm{P}(\text{spin}=3)=\frac{72}{360}=\frac{1}{5}=\frac{2}{10}$ $\mathrm{P}(\text{spin}=4)=\frac{36}{360}=\frac{1}{10}$	The Table of Outcomes and calculation of individual probabilities is preparation work, useful for subsequent calculations. It is not essential and hence not given any credit for this question.
Identify and list the possible outcomes that meet the stated criteria $X=6$ and calculate associated probability values for each outcome	**(a)**	$\mathrm{P}(X=6)$ $=\mathrm{P}(\text{spin}_1=2,\text{spin}_2=2)+\mathrm{P}(\text{spin}_1=2,\text{spin}_2=4)+\mathrm{P}(\text{spin}_1=4,\text{spin}_2=2)$ $=\left[\left(\frac{3}{10}\right)\left(\frac{3}{10}\right)\right]+\left[\left(\frac{3}{10}\right)\left(\frac{1}{10}\right)\right]+\left[\left(\frac{1}{10}\right)\left(\frac{3}{10}\right)\right]$ $=0.15$	Most students were able to list out the cases for $X=6$ and substitute the correct probabilities.

Analyse and identify all possible cases and calculate all associated probabilities	**(b)**	$\mathrm{P}(X=3)$ $=\mathrm{P}(\text{spin}_1=1,\text{spin}_2=1)+\mathrm{P}(\text{spin}_1=1,\text{spin}_2=2)+\mathrm{P}(\text{spin}_1=2,\text{spin}_2=1)$ $=\left[\left(\frac{4}{10}\right)\left(\frac{4}{10}\right)\right]+\left[\left(\frac{4}{10}\right)\left(\frac{3}{10}\right)\right]+\left[\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)\right]$ $=0.4$ $\mathrm{P}(X=4)$ $=\mathrm{P}(\text{spin}_1=1,\text{spin}_2=3)+\mathrm{P}(\text{spin}_1=3,\text{spin}_2=1)$ $=\left[\left(\frac{4}{10}\right)\left(\frac{2}{10}\right)\right]+\left[\left(\frac{2}{10}\right)\left(\frac{4}{10}\right)\right]$ $=0.16$ $\mathrm{P}(X=5)$ $=\mathrm{P}(\text{spin}_1=1,\text{spin}_2=4)+\mathrm{P}(\text{spin}_1=4,\text{spin}_2=1)+\mathrm{P}(\text{spin}_1=2,\text{spin}_2=3)+\mathrm{P}(\text{spin}_1=3,\text{spin}_2=2)$ $=\left[\left(\frac{4}{10}\right)\left(\frac{1}{10}\right)\right]+\left[\left(\frac{1}{10}\right)\left(\frac{4}{10}\right)\right]+\left[\left(\frac{3}{10}\right)\left(\frac{2}{10}\right)\right]+\left[\left(\frac{2}{10}\right)\left(\frac{3}{10}\right)\right]$ $=0.2$ $\mathrm{P}(X=7)$ $=\mathrm{P}(\text{spin}_1=3,\text{spin}_2=4)+\mathrm{P}(\text{spin}_1=4,\text{spin}_2=3)$ $=\left[\left(\frac{2}{10}\right)\left(\frac{1}{10}\right)\right]+\left[\left(\frac{1}{10}\right)\left(\frac{2}{10}\right)\right]$ $=0.04$ $\mathrm{P}(X=9)$ $=\mathrm{P}(\text{spin}_1=3,\text{spin}_2=3)$ $=\left[\left(\frac{2}{10}\right)\left(\frac{2}{10}\right)\right]$ $=0.04$	A Table of Outcomes will be useful to identify all the possible values of X. Most students who attempted this part were able to identify all the possible of X correctly. Please read the question carefully. The question did ***not*** specifically ask for a Probability Distribution ***Table***, hence it was not necessary to present the probabilities in a table format.

		$\mathrm{P}(X=12)$ $=\mathrm{P}(\text{spin}_1=4,\text{spin}_2=4)$ $=0.01$ **Alternative presentation format (Probability Distribution Table)** x: 3 | 4 | 5 | 6 | 7 | 9 | 12 $\mathrm{P}(X=x)$: $\frac{40}{100}=\frac{2}{5}=0.4$ | $\frac{16}{100}=\frac{4}{25}=0.16$ | $\frac{20}{100}=\frac{1}{5}=0.2$ | $\frac{15}{100}=\frac{3}{20}=0.15$ | $\frac{4}{100}=\frac{1}{25}=0.04$ | $\frac{4}{100}=\frac{1}{25}=0.04$ | $\frac{1}{100}=0.01$	
Calculate conditional probability	**(c)**	$\mathrm{P}(\text{Score}<10 \mid \text{Customer wins a prize})$ $=\mathrm{P}(X<10 \mid X>6)$ $=\frac{\mathrm{P}(X<10 \cap X>6)}{\mathrm{P}(X>6)}$ $=\frac{\mathrm{P}(X=7)+\mathrm{P}(X=9)}{\mathrm{P}(X=7)+\mathrm{P}(X=9)+\mathrm{P}(X=12)}$ $=\frac{(0.04)+(0.04)}{(0.04)+(0.04)+(0.01)}$ $=\frac{8}{9}$ or 0.889 (3 s.f.)	This question was a conditional probability question. Many students did not interpret the question correctly. There were some students who included $X=6$, i.e. using $X \geq 6$ instead of $X>6$ for the condition of winning a prize.

Q6 Correlation and Linear Regression [LL[]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Sketch a scatter diagram with the aid of a G.C. Key skills: - label axes - Check correct number of points plotted - label minimum and maximum x and y values on the scatter plot	**(a)**		It was important to demonstrate the key skills stated in the assessment objective. Most students did not label the end points. Some students labeled the axes wrongly. For this question, temperature t (°C) was the independent variable, hence it should be the x-axis
Calculate the product moment correlation coefficient with the aid of a G.C.		$r = 0.9474557$ $= 0.947$ (3 s.f.)	Most students were able calculate the correlation coefficient correctly using the GC. There were a small number of students who used the formula to calculate, which was not necessary for this question.
Explain suitability of a linear model using both the product moment correlation coefficient and scatter diagram.		Although the product moment correlation coefficient $r = 0.947$ is close to +1, which suggests a strong, positive, linear relationship, The scatter diagram indicates that as t increases, s increases at a decreasing rate i.e. the scatter diagram shows the points appear to lie on a curve rather than a straight line, so a linear model may not model the relationship well.	The important concept to understand was that ***both*** the r-value and scatter plot must be considered together to reach a conclusion.
Perform linearization of a non-linear model and find the equation of the linear regression line with the aid of a G.C.	**(b)**	For $s = a \ln t + b$,  $a = 436.107$ (3 d.p.) $b = -923.838$ (3 d.p.)	Most students who attempted this part were able to use the GC correctly to calculate the correct a and b values. However, there were quite a number who did not read the question correctly and failed to give their answers to the required accuracy.

Calculate the product moment correlation coefficient of the transformed relationship with the aid of a G.C.	**(c)**	$r = 0.967849 = 0.968 (3 \text{ s.f.})$	Most students who attempted this part were able to use the GC to calculate the new *r*-value.
Compare and contrast between different models using suitable indicators (the *r*-value)	**(d)**	$s = a \ln t + b$ is a better model as the *r*-value is closer to 1.	Most students were able to explain their choice using the key phrase $\lvert r \rvert$ is closer to 1. Simply stating one new *r*-value being "greater than" the other is not sufficient.

Use a linear regression line for estimation. Discern whether interpolation or extrapolation has been performed and the reliability of the estimate	**(e)**	When $t = 38$, $s = 436.107 \ln(38) - 923.8376$ $s = 662.53655$ Sales = 66,254 units Not reliable since 38°C is out of the data range hence extrapolation was performed.	Most students who attempted this part were able to calculate the estimate correctly using the regression line from **(b)** However, many failed to consider that in the context of the question, sales was measured in hundreds of units Most students were able to identify that 38°C was out of the data range and hence extrapolation was carried out, leading to an unreliable estimate. The key phase "extrapolation" was necessary. There were a small number of students who tried to explain correlation and causation, but this explanation was not addressing the reliability of the estimate.

Q7 Permutations and Combinations & Probability [KP]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Number of selections involving restrictions.	**(a)(i)**	No. of ways $= {}^{12}C_3 \times 3! = 1320$	Students commonly compute as ${}^{12}C_3$, omitting the need for arrangement as it consists of 3 positions, chairperson, vice-chairperson and secretary.
	(a)(ii)	**Method ①:** No. of ways $= \underbrace{{}^{12}C_2 \times {}^{6}C_1 \times 3!}_{\text{Select 2 girls from 12 \& 1 boy from 6 followed by arrangement}} + \underbrace{{}^{12}C_1 \times {}^{6}C_2 \times 3!}_{\text{Select 1 girl from 12 \& 2 boys from 6 followed by arrangement}}$ $= 3456$ **Method ②:** No. of ways $= \underbrace{{}^{18}C_3 \times 3!}_{\text{No restriction}} - \underbrace{{}^{12}C_3 \times 3!}_{\text{Select 3 girls from 12 followed by arrangement}} - \underbrace{{}^{6}C_3 \times 3!}_{\text{Select 3 boys from 6 followed by arrangement}}$ $= 3456$	Students commonly omit the need for arrangement as it consists of 3 positions, chairperson, vice-chairperson and secretary.
Probabilities involving arrangement of objects in a circle, where certain objects are separated.	**(b)**	**Method ①:** Required probability $= \dfrac{(15-1)! \times {}^{15}C_3 \times 3!}{(18-1)!}$ $= \dfrac{91}{136}$	A number of students did not realised that this part of the question onwards require probability. Many continued to find number of ways. There were students who were confused over the multiplication

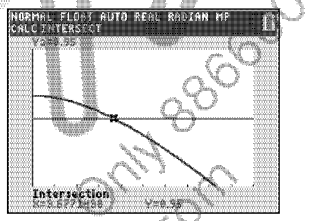
		Method ②: [Not Recommended] $\text{Required probability} = 1 - \underbrace{\frac{(16-1)!\times 3!}{(18-1)!}}_{\text{chairperson, vice-charperson and secretary together}} - 3\underbrace{\left[\frac{(15-1)!\times {}^{15}C_2\times 2!\times 2!}{(18-1)!}\right]}_{\text{any 2 together and 1 separate}}$ $= \frac{91}{136}$	and addition principal. Students who used Method ② often excluded the cases where any 2 can be together and 1 separated.
Probabilities involving arrangement of objects in a circle, where there is a given restriction.	**(c)**	**Method ①: [Arrange boys first followed by the girls]** $\text{Required probability} = \frac{(6-1)!\times 12!}{(18-1)!}$ $= \frac{1}{6188}$ **Method ②: [Arrange girls first followed by the boys]** $\text{Required probability} = \frac{\left((6-1)!\times \frac{{}^{12}C_2\times {}^{10}C_2\times {}^{8}C_2\times {}^{6}C_2\times {}^{4}C_2\times {}^{2}C_2}{6!}\times 2^6\right)\times 6!}{(18-1)!}$ $= \frac{1}{6188}$	Most students were able to arrange the boys or girls first successfully but struggled to complete the remaining arrangement for the other gender.

Q8 Binomial Distribution [KP]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
State the conditions under which the binomial distribution is a suitable model.	**(a)**	The probability of an orange being rotten is constant at p%. The event of an orange being rotten is independent of the event of any other orange being rotten.	This was not well done. Some students stated the following conditions of binomial distribution when they were implied in the question. • There is a fixed number of trials. • There are two mutually exclusive outcomes – an orange being rotten and an orange being non-rotten. The above conditions were covered in the description given in the question, so they were not assumptions that need to be made. A significant percentage of students mixed up the 2 assumptions and stated "The probability of an orange being rotten is independent of any other orange being rotten." The words 'probability' and 'independent' should not be used in the same sentence. Some students seemed to think that the assumptions only applied to the oranges in the packet (sample). The

			assumptions should be applied for all the oranges in the bin (population). Some students mixed up the assumptions for binomial distribution with the conditions to form a random sample. Words such as 'choose', 'select', 'pick' and 'find' should not be used. <u>Some Common Mistakes</u> • Use of 'trial' and 'population' without explanation in context • Failed to use appropriate Mathematics terminologies such as 'probability' (not 'chance') and 'independent' (not 'will not affect'). • The percentage $p\%$ is constant. ($p\%$ needs to be explained in context) • A rotten orange is independent of any other rotten orange. (it should be about whether an orange being rotten is independent of any other orange being rotten).
Probability for a binomial distribution.	**(b)(i)**	Let X be the random variable denoting the number of rotten oranges out of 10 oranges. (defined by the question)	This part was not well done.

		$X \sim \mathrm{B}(10, 0.2)$ $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$ $= 0.6241903616$ $= 0.624 \ \ (\text{to 3 s.f.})$	Some students were not able to formulate the question mathematically. The phrase 'at least 2' means that 2 is included in the consideration. A significant percentage of students were not able to re-express $\mathrm{P}(X \geq 2)$ into a form which can be calculated using G.C. command 'binomcdf'. Common mistakes • $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 2)$ • $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X = 1)$ $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X = 0)$

Determine expectation of a binomial distribution in a given context.	**(b)(ii)**	Expected number of packets of oranges that contains more than 1 rotten orange $=100\left[\mathrm{P}\left(X>1\right)\right]$ $=100\left[1-\mathrm{P}\left(X\le 1\right)\right]$ $=62.41903616$ **Method 1** Expected profit when all the packets of oranges are sold $=2\left(100\right)=200$ For the store manager to have a net profit, Expected loss < Expected profit $62.41903616d<200$ $d<3.204150726$ $d<3.20\ \ (\text{to 2 d.p.})$ $\therefore 0<d<3.20$ **Method 2** Total profit $=\left(100-62.41903616\right)\left(2\right)+62.41903616\left(2-d\right)$ $=200-62.41903616d$ For the store manager to have a net profit, Total profit >0 $200-62.419d>0$ $d<3.20$ (to 2 d.p.) $\therefore 0<d<3.20$	This part was not well done. Some students were not able to find the probability that a packet contains more than 1 rotten orange. The phrase 'more than 1' is equivalent to 'at least 2'. Students should use at least 4 decimal places in the intermediate steps to ensure the final answer is accurate up to 2 decimal places. One common mistake made by students was to find the mode of the distribution of the number of packets of oranges that contain more than 1 rotten orange. It is important to realize that the mode is not the same as the expected number.

Determine unknown probability p when given certain conditions.	**(c)**	Let X be the random variable denoting the number of rotten oranges out of 10 oranges. (defined by the question) $X \sim \mathrm{B}(10, 0.01p)$ $\mathrm{P}(X \le 1) = 0.95$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.95$ $^{10}\mathrm{C}_0 (0.01p)^0 (1-0.01p)^{10} + {}^{10}\mathrm{C}_1 (0.01p)^1 (1-0.01p)^9 = 0.95$ $(1-0.01p)^{10} + 0.1p(1-0.01p)^9 = 0.95$ Using G.C., $p = 3.6771438$ $= 3.68$ (to 3 s.f.) 	This part was poorly attempted. A significant percentage of students did not realize that p was previously defined at the start of the question. They had thought that they were to find a probability of an event. One of the commonly seen mistakes was the use of p instead of $0.01p$. The correct notation for p% should be $0.01p$ or $\frac{p}{100}$ (not $0.0p$). Another common mistake was to state the value of p as 3.68% (= 0.0368) instead of 3.68.

Q9 Normal Distribution [LL]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Calculate probabilities from a Normal Distribution with given mean and variance using the GC. Consider the possible outcomes (i.e. cases) which satisfy the given criteria.	**(a)**	Required probability $= \left[\mathrm{P}(A<140)\right]^2 \times \mathrm{P}(A>170) \times \frac{3!}{2!}$ $= \left[\mathrm{P}(A<140)\right]^2 \times \mathrm{P}(A>170) \times 3$ $= (0.3341176)^2 \times (0.26015833) \times 3$ $= 0.087128$ $= 0.0871 (3 \text{ s.f})$	Poorly done. Students either missed out the square in $\left[\mathrm{P}(A<140)\right]^2$ or $\frac{3!}{2!}$. Some students even wrote $\mathrm{P}(A_1 + A_2 < 280)$ for the case of "two of the apples each has mass less than 140g". They did not realize that the masses of the apples are independent of one another, as stated in the question.
Calculate the means and variances of Linear combination of independent Normal Variables $aX \pm bY$	**(b)**	Let A be the random variable denoting the mass of an apple from the supermarket. Let G be the random variable denoting the mass of a guava from the supermarket. $A \sim N(152, 28^2)$ and $G \sim N(268, 43^2)$ $X = A_1 + A_2 + A_3 + A_4 + A_5$ $X \sim \mathrm{N}(5\times 152, 5\times 28^2)$ $X \sim \mathrm{N}(760, 3920)$ $Y = G_1 + G_2 + G_3$ $Y \sim \mathrm{N}(3\times 268, 3\times 43^2)$ $Y \sim \mathrm{N}(804, 5547)$ $X - Y \sim \mathrm{N}(760-804, 3920+5547)$ $X - Y \sim \mathrm{N}(-44, 9467)$	Quite a number of students interpreted the "total mass of five randomly chosen apples" as "5A" and the "total mass of three randomly chosen guavas as "3G", leading them to get the wrong mean and variance . Another careless mistake is taking $\mathrm{Var}(A) = 28$ and $\mathrm{Var}(G) = 43$, missing out the squares altogether, when only the **standard deviation** of each random variable were given. Otherwise, most students were able to get the answer.

		$\mathrm{P}(X<Y)=\mathrm{P}(X-Y<0)$ $=0.6744435$ $=0.674\ (\text{to 3 s.f.})$	
To interpret and solve probabilities involving modulus of Normal distributions.	**(c)**	$F=A_1+A_2+A_3+G_1+G_2$ $F\sim \mathrm{N}\left(3\times 152+2\times 268, 3\times 28^2+2\times 43^2\right)$ $F\sim \mathrm{N}\left(992, 6050\right)$ Given $\mathrm{P}(\lvert F-992\rvert<m)=0.95$ $\mathrm{P}(-m<F-992<m)=0.95$ $\mathrm{P}(992-m<F<992+m)=0.95$ **<u>Method ①:</u> Using right Tail** $992+m=1144.449$ $m=152.449$ $m=153\ (3\text{ s.f.})$ **<u>Method ②:</u> Using left tail** $992-m=839.551$ $m=152.449$ $m=153\ (3\text{ s.f.})$	A lot of students do not know how to interpret the question. They took "**mean mass** of the Family Packs" as "*sample mean* of the Family Packs", which will not give the solution to what the question asked for. Some students did not know how to break up $\lvert F-992\rvert<m$ into $992-m<F<992+m$ and with all sorts of other combinations which cannot derive the answer. For the question, it is not advisable to use GC table to find the answer to the value of m since it is not an integral value. Students should just use inverse Normal with the GC to find the value that is required.
To interpret the question and perform necessary conversion of units, and calculate the means and variance of the new random	**(d)**	**<u>Method ①:</u> Convert weight from grams to kilograms to use selling price in \$/kg given in question** $F\sim N\left(992, 6050\right)$ (in g)	Common mistake for method one: students only divided by 1000 to 6050 for the variance instead of dividing by 1000^2.

variable in the appropriate units		$$F' \sim N\left(\frac{992}{1000}, \frac{6050}{1000^2}\right) \text{ (in kg)}$$ $$F' \sim \mathrm{N}(0.992, 0.00605)$$ Let C be the cost of a Family Pack (\$/kg). $C = 5F'$ $$C \sim \mathrm{N}(5 \times 0.992, 5^2 \times 0.00605)$$ $$C \sim \mathrm{N}(4.96, 0.15125)$$ $\mathrm{P}(C < 5) = 0.54096$ $= 0.541$ (to 3 s.f.) **<u>Method ②:</u> Convert selling price from \$/kg to \$/g** Let C be the cost of a Family Pack in \$/gram and F be the total mass of a Family Pack in grams (from **(iii)**). Selling price of Family pack = \$5/kg = \$0.005/g $C' = 0.005F$ $$C' \sim \mathrm{N}(0.005 \times 992, (0.005)^2 \times 6050)$$ $$C' \sim \mathrm{N}(4.96, 0.15125)$$ $\mathrm{P}(C < 5) = 0.54096$ $= 0.541$ (to 3 s.f.) **<u>Method ③:</u> Convert random variable from price to weight in grams** $$F \sim N(992, 6050) \text{ (in g)}$$ Let C be the cost of a Family Pack (\$/kg).	Students who did by method 2 or 3 were able to get the answer.

		$C = \frac{5}{1000}F$ $\mathrm{P}(C < 5)$ $= \mathrm{P}\left(\frac{5}{1000}F < 5\right)$ $= \mathrm{P}(F < 1000)$ $= 0.54096$ $= 0.541 \text{ (to 3 s.f.)}$	

Q10 Hypothesis Testing [LL]			
Assessment Objectives	**Solution**		**Examiner's Feedback**
Calculate unbiased estimates of the population mean and variance when given consolidated data.	**(a)**	Unbiased estimates of the population mean $= \frac{\sum(x-650)}{n} + 650$ $= -\frac{34.39}{50} + 650$ $= 649.3122$ Unbiased estimates of the population variance, s^2 $= \frac{1}{n-1}\left(\sum(x-650)^2 - \frac{\left(\sum(x-650)\right)^2}{n}\right)$ $= \frac{1}{49}\left(22769.98 - \frac{(-34.39)^2}{50}\right)$ $= \frac{89621}{6087}$ or 464.21 $= 464$ (to 3 s.f.)	Unfortunately, this part was not well done by candidates. Candidates did not apply the formulae accurately, especially finding unbiased estimate of the population variance where it can be easily found in MF26. For unbiased estimates of population mean, candidates did not add 650, hence their answer ends up to a negative value but did not realise something is wrong. The unbiased estimate of population mean is exact at 649.3122. Candidates **should not round up to 3 s.f.,** making it less exact.
Perform Hypothesis Testing based on the requirements stated in the question. The particular population in this question is not explicitly stated as being normally distributed, hence application of CLT is necessary.	**(b)**	$H_0 : \mu = 650$ $H_1 : \mu < 650$, μ is the population mean travelling distance on a single charge Under H_0, Since sample size, n = 50 is sufficiently large, $\overline{X} \sim N\left(650, \frac{464.21}{50}\right)$ approximately by CLT	This topic is poorly done. It is obvious whether candidates study the topic by looking at their presentation, some clearly did not know what was required for this question. Some candidates got the alternative hypothesis testing wrong. It should be a left tail test. If Vesla (representing H_0)

Distribution of test statistic $Z = \dfrac{\overline{X} - 650}{\sqrt{\dfrac{464.21}{50}}} \sim \mathrm{N}(0,1)$

Test statistic, $z = \dfrac{649.3122 - 650}{\sqrt{\dfrac{464.21}{50}}} = -0.22573 \approx -0.226$ (3 s.f.)

Critical value method	p-value method
At 5% level of significance, we reject H_0 if $z_{\text{test}} \leq -1.64485$.	At 5% level of significance, we reject H_0 if p-value ≤ 0.05.
Since $z_{\text{test}} \leq -1.64485$, we do not reject H_0 and conclude that there is insufficient evidence at the 5% level of significance that the car manufacturer has overstated the travelling distance on a single charge.	p-value $= 0.4107056 \approx 0.411$ (3 s.f.) Since p-value $= 0.411 > 0.05$, we do not reject H_0 and conclude that there is insufficient evidence at the 5% level of significance that the car manufacturer has overstated the travelling distance on a single charge.

overstated its claim i.e. the Car Reviewer thinks that the mean is too high. Therefore, the car reviewer (representing H_1) needs to test and he should test for a lower mean instead.

The question mentioned "define any symbols you use", candidates should define the population mean μ in the context of the question.

Candidates should not write $X \sim \mathrm{N}(650, 464.21)$ since X is not stated to be normal. They should write the distribution of $\overline{X}$ along with the conditions where CLT is applied.

Since the question also mentioned "work out the test statistic", then the test statistic should be shown.

Candidates need to know the difference between Z and z. Z is a distribution written in terms of $\overline{X}$, while z is a test statistic which is a value and all values can be substituted inside, giving rise to -0.22573 .

			They can use the critical value method or p-value method to give the conclusion. There were a significant number of candidates who got the correct p-value = 0.411 concluded that this is less than 0.05. They probably thought 4 is less than 5 subconsciously, please avoid this careless mistake.
Apply the conditions for using CLT to the context of the problem	**(c)**	The MeTube car reviewer **needs** to apply Central Limit Theorem (CLT) because the **distribution of the travelling distance on a single charge is unknown (i.e. not normally distributed).** With the sample size of 50 (large), CLT can be applied and the <u>mean</u> travelling distance on a single charge is <u>approximately</u> normally distributed.	There are 2 conditions to be satisfied before CLT can be applied: 1. Distribution is not normal / unknown. 2. Sample size is sufficiently large. Many candidates could state the second condition (n is large) but not the first, and also did not answer the question properly if CLT needs to be used. With regards to the second statement of the answer, candidates are still weak in using the correct terms. When sample size is large, CLT is used/ applied. Candidates should not write CLT is "assumed" or "approximated". (Please avoid)

			Also, CLT is applied so that the distribution of the **mean** travelling distance i.e. $\overline{X}$ is approximately normal. It does not equate to the distribution of the travelling distance i.e. X is approximately normal. X still remains to be an unknown distribution. Hence the word "**mean"** is crucial.
Suggest the Alternative Hypothesis H_1 based on the context of the question	**(d)**	The TokTik car reviewer should use a 2-tail test since travelling distance on a single charge can either be **more than or less than 650 km.**	Other acceptable keywords include: differs from 650km. Some candidates who wrote "because the car reviewer wants to test if the claim is true" is not accepted. All hypothesis tests are testing whether the claim is true – it did not address why a 2 tailed test should be used instead of a one-tail test. Some state the correct test without explaining. No credit is awarded since the question is not addressed.
Demonstrate understanding of the conditions under which a Hypothesis Test can be conducted. In the absence of such conditions being stated explicitly in the question, suitable assumptions need to be made in	**(e)**	The TokTik car reviewer needs to assume that the travelling distance on a single charge is normally distributed. He also needs to assume that the observations of travelling distance on a single charge are independent. Accept: Assume that the unbiased estimate of the population variance is a	Some candidates mentioned "normally distributed" and "independent" but their phrasing distorted the accurate meaning of the required assumptions.

order for the test to be conducted.		good estimate of the population variance.	It is incorrect to say the tests/ samples are normally distributed. It should be the distribution of ___. For independence, some candidates used the independence condition in the binomial distribution explanation which is not correct. The event of interest here is the "observations (or results) of the travelling distance", not so much about the car. There were also many strange assumptions made which are unacceptable. More practice is required for all candidates to improve the phrasing of such questions in future.