1 It is given that P, Q and R are the angles of a triangle.

(a) Show that $\cos P = -\cos(Q + R)$. [2]

$\cos P$	$= \cos[180° - (Q + R)]$	[M1 – for replacing]
	$= \cos 180° \cos(Q + R) + \sin 180° \sin(Q + R)$	
	$= -\cos(Q + R) + 0$	[A1 – apply addition formula &
	$= -\cos(Q + R)$	[evaluate to arrive at result]

Accept supplementary angles:

$\cos P$	**$= -\cos(180° - P)$**	**M1**
	$= -\cos(Q + R)$	**A1**

(b) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

$\cos P$	$= -\cos(45° + 60°)$	
	$= -\cos 45° \cos 60° + \sin 45° \sin 60°$	[M1 – correct use of formula expansion]
	$= -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$	[M1 – correct special angles trigo ratios]
	$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$	[A1]

2 Baking powder is poured onto a flat surface at a constant rate of 2π cm^3s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

Vol. of cone, $V = \frac{1}{3}\pi r^2(18r)$

$= 6\pi r^3$

$\frac{\mathrm{d}V}{\mathrm{d}r} = 18\pi r^2$ [B1]

After 3 seconds, $V = 6\pi$

$6\pi r^3 = 6\pi$ [M1 - finding corresponding r]

$r = 1$ [A1]

$\frac{\mathrm{d}V}{\mathrm{d}t} = \left.\frac{\mathrm{d}V}{\mathrm{d}r}\right|_{r=1} \times \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{r=1}$ [M1 - connected rate of change]

$2\pi = 18\pi(1)^2 \times \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{r=1}$

$\left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{r=1} = \frac{1}{9}$

The rate of change required is $= \frac{1}{9}$ cm/s. [A1 o.e.]

3 **(a)** Determine the set of values of m for which the equation $2x^2+4x+2m=6mx-2$ has real roots. [4]

$$2x^2+(4-6m)x+2m+2=0$$

$$b^2-4ac\geq 0$$ [M1 - correct Discriminant]

$$(4-6m)^2-4(2)(2m+2)\geq 0$$

$$16-48m+36m^2-8(2m+2)\geq 0$$

$$36m^2-64m\geq 0$$ [M1 – simplification in factors]

$$m(9m-16)\geq 0$$

$$m\leq 0 \quad or \quad m\geq\frac{16}{9}$$ [A2, minus 1 mark if inequality sign is wrong due to earlier wrong D sign]

(b) Hence state what can be deduced about the curve $y=2(x+1)^2$ and the line $y=6x-2$. Justify your statement. [2]

$2x^2+4x+2=6x-2$

By comparing with (a), m = 1 [B1 – correct m value]

When m = 1, it is not within the set of values of m for which there will be real roots, hence the curve will not meet the line/ the curve will not cut the line. [B1]

4 **(a)** Show that $\frac{d}{dx}(\ln(\cos x)) = -\tan x$. [2]

$\frac{d}{dx}(\ln(\cos x)) = \frac{1}{\cos x} \times \frac{d}{dx}(\cos x)$	[M1 – show working]
$= \frac{-\sin x}{\cos x} = -\tan x$	[A1 – show fraction]

(b) Differentiate $x \tan x$ with respect to x. [2]

$\frac{d}{dx} x \tan x = x \sec^2 x + \tan x$	[M1 – show product rule] [A1 – correct ans for both]

(c) Using the results from part **(a)** and **(b)**, find $\int x \sec^2 x \, dx$ and hence show that $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

From (b), $\int (x\sec^2 x + \tan x) \, dx = x \tan x + C$	[M1 – use part (b), 'C' must be seen]
$\int x\sec^2 x \, dx + \int \tan x \, dx = x \tan x + C$	
$\int x\sec^2 x \, dx = x \tan x - \int \tan x \, dx + C$ $= x \tan x + \ln(\cos x) + C$	[M1 – proper integration and final correct ans, 'C' must be seen]
$\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \left[x \tan x + \ln(\cos x)\right]_0^{\frac{\pi}{4}}$	[minus 1 mark if "C" is not seen]
$= \frac{\pi}{4}\tan\frac{\pi}{4} + \ln\left(\cos\frac{\pi}{4}\right) - 0 - \ln 1$	
$= \frac{\pi}{4} + \ln\left(\frac{1}{\sqrt{2}}\right)$	[M1 – proper evaluation]
$= \frac{\pi}{4} + \ln 2^{-\frac{1}{2}}$	
$= \frac{\pi}{4} - \frac{1}{2}\ln 2$	[A1]

5 **(a)** In the expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [3]

$$(2+x)^n = 2^n + \binom{n}{1}2^{n-1}x + \binom{n}{2}2^{n-2}x^2 + \ldots$$

$$\binom{n}{2}2^{n-2} = 2\binom{n}{1}2^{n-1}$$ [M1 – correct coeff]

$$\frac{n(n-1)}{2}\left(2^{n-2}\right) = 2n\left(2^{n-1}\right)$$

$$\frac{n(n-1)}{2}\left(2^n \times 2^{-2}\right) = 2n\left(2^n \times 2^{-1}\right) \quad (n \neq 0)$$ [M1 – simplify]

$$\frac{n-1}{2(4)} = 1$$

$$n - 1 = 8$$

$$n = 9$$ [A1 – with rejection]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x - \frac{1}{4x^4}\right)^{15}$. [4]

general term

$$= \binom{15}{r}(2x)^{15-r}\left(-\frac{1}{4x^4}\right)^r$$

$$= \binom{15}{r}2^{15-r}x^{15-r}\left(-\frac{1}{4}\right)^r x^{-4r}$$ [M1]

$$= \binom{15}{r}2^{15-r}\left(-\frac{1}{4}\right)^r x^{15-5r}$$ [M1 - gather x and let power = 0]

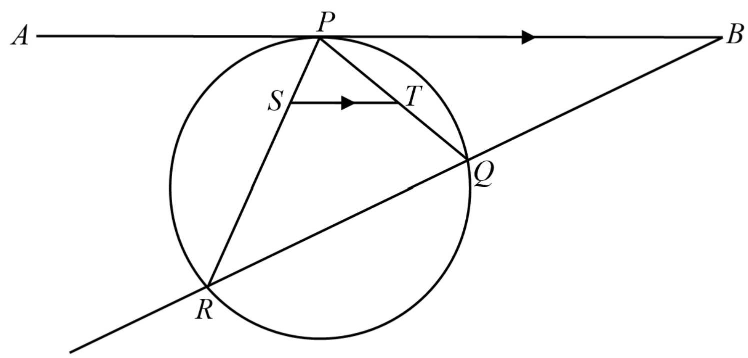
$$15 - 5r = 0$$

$$r = 3$$ [M1 for r value]

$$\text{value} = \binom{15}{3}2^{12}\left(-\frac{1}{4}\right)^3 = -29120$$ [A1]

Max 2 marks if able to gather 15 – 5r for exponent of 'x' and equate to zero with correct r value.

6



The diagram shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(a) triangle PST is similar to triangle PQR, [3]

$\angle SPT = \angle QPR$ (common angle) [M1]

$\angle PST = \angle SPA$ (alt. $\angle$s, AB // ST) **OR** $\angle PTS = \angle TPB$ (alt. $\angle$s, AB // ST)

$= \angle PQR$ (Alt. Segment Thm) $= \angle PRQ$ (Alt. Segment Thm) [M1]

$\therefore$ triangle PST is similar to triangle PQR. [A1]

(2 pairs of corresponding angles are equal)

(b) $PQ \times PT = PR \times PS$, [2]

From above result, $\dfrac{PQ}{PS} = \dfrac{PR}{PT}$ [M1]

$\Rightarrow \quad PQ \times PT = PR \times PS$ [A1]

(c) Determine if $STQR$ is a cyclic quadrilateral. [4]

$QTS = 180° - \angle PTS$ (adj. $\angle$s on a st. line)

$= 180° - \angle QRS$ (from (a) result) [M1]

So $\angle QTS + \angle QRS = 180°$ [M1]

$\angle RST + \angle RQT = 360° - (\angle QTS + \angle QRS)$ ($\angle$ sum of a quadrilateral)

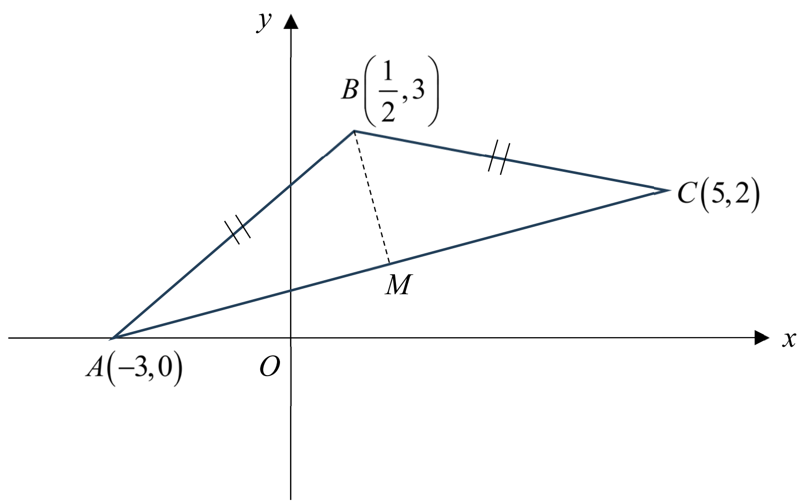
$= 360° - 180°$

$= 180°$ [M1]

By converse of angles in opposite segment, $STQR$ is a cyclic quadrilateral and all four vertices lie on the circumference of a circle.

[A1 – with correct reason, accept even if no mention of four vertices]

7



The diagram shows an isosceles triangle ABC in which $A(-3,0)$, $B\left(\frac{1}{2},3\right)$ and $C(5,2)$. M is the foot of perpendicular from B to AC.

(a) Find the coordinates of M. [1]

$M = \left(\frac{-3+5}{2}, \frac{0+2}{2}\right) = (1, 1)$ [B1]

(b) Find the equation of the perpendicular bisector of AC. [2]

Gradient of $AC = \frac{2-0}{5-(-3)} = \frac{1}{4}$

Gradient of perpendicular BM = – 4 [M1 – use $m \times m_{\perp} = -1$]

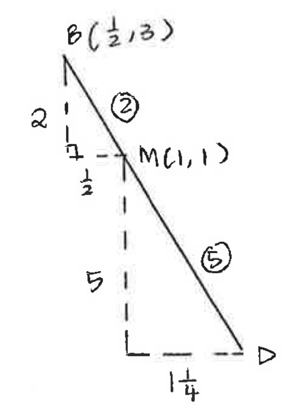
or $m_{BM} = \frac{3-1}{\frac{1}{2}-1} = -4$

Equation of perpendicular bisector:

$y-1=-4(x-1)$

$y=-4x+5$ [A1]

(c) Given that *ABCD* is a kite with $BM = \frac{2}{7}BD$, find the coordinates of *D*. [3]



Correct ratio: 2 : 5

$\frac{1}{2} : 1\frac{1}{4}$ M1

Coordinates of D

$= \left(1 + \frac{1}{4}, 1 - 5\right)$ M1

$= \left(2\frac{1}{4}, -4\right)$ A1

Alternative: using vectors

$\overrightarrow{BM} = \frac{2}{7}\overrightarrow{BD}$

$\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} = \frac{2}{7}\left(\overrightarrow{OD} - \overrightarrow{OB}\right)$ [M1 - position vectors]

$\overrightarrow{OD} = 3.5\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2.25 \\ -4 \end{pmatrix}$ [M1]

D$= \left(2\frac{1}{4}, -4\right)$ [A1]

(d) Find the area of the kite *ABCD*. [2]

Area of *ABCD* $= \frac{1}{2}\begin{vmatrix} -3 & 2\frac{1}{4} & 5 & \frac{1}{2} & -3 \\ 0 & -4 & 2 & 3 & 0 \end{vmatrix}$

M1 (ecf 1 – correct method)

$= \frac{1}{2}(31.5 + 28)$ (anti-clockwise)

$= 29.75$ units^2 (or $\frac{119}{4}$) A1

(accept 29.7 with 3 s.f. if using other method such as Pythagoras'

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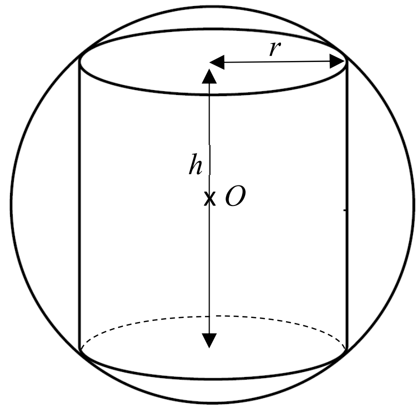


A prototype consists of a cylindrical container of height h cm and radius r cm inscribed in a hollow sphere with centre O.

The sphere has a surface area of 6400π cm^2 and both the sphere and container have negligible thickness.

(a) Show that the volume of the cylinder container, V cm^3, is given by [3]

$$V = 2\pi r^2 \sqrt{1600 - r^2}.$$

$4\pi R^2 = 6400\pi$	[M1 – find radius R]
$R = 40$ (radius R of sphere)	
By Pythagoras' Theorem,	
$r^2 + \left(\frac{h}{2}\right)^2 = 40^2$	[M1]
$h^2 = 4\left(1600 - r^2\right)$	
$h = \sqrt{4\left(1600 - r^2\right)}$	
$h = 2\sqrt{1600 - r^2}$	
$V = \pi r^2 h$	
$= 2\pi r^2 \sqrt{1600 - r^2}$	[A1]

(b) Given that r can vary, find the value of r for which the volume V is stationary. [5]

$$\frac{dV}{dr} = (2\pi r^2)\times\frac{1}{2}\left(1600-r^2\right)^{-\frac{1}{2}}\times(-2r)+\sqrt{1600-r^2}\times 2\times 2\pi r$$ [M2 – correct differentiation using product rule, deduct M1 if either term is incorrect]

$$\frac{dV}{dr} = 4\pi r\sqrt{1600-r^2}-\frac{2\pi r^3}{\sqrt{1600-r^2}}$$

For stationary values, $\frac{dV}{dr}=0$

$$4\pi r\sqrt{1600-r^2}-\frac{2\pi r^3}{\sqrt{1600-r^2}}=0$$ [M1 – equate to zero]

$$\frac{2\pi r}{\sqrt{1600-r^2}}\left[2\left(1600-r^2\right)-r^2\right]=0$$ [M1 – simplify and show this or the next line working]

$$\frac{2\pi r}{\sqrt{1600-r^2}}\left[3200-3r^2\right]=0$$

$r=0$ (reject) or $3200-r^2=0$

$r=\frac{40\sqrt{6}}{3}=32.7\text{ cm}$ [A1]

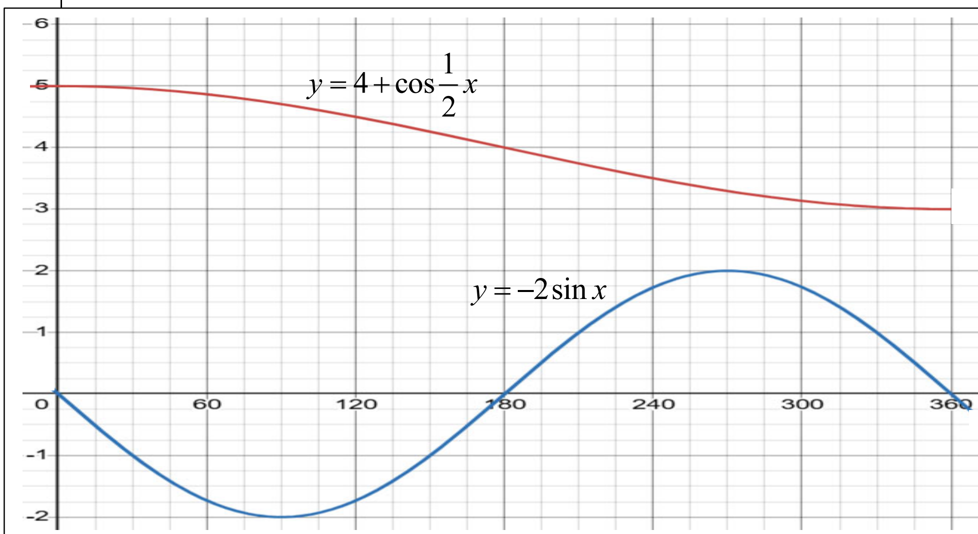
(c) A scientist plans to launch this prototype into outer space carrying as much fuel as possible. Explain whether the prototype can satisfy the scientist's requirement. [2]

x	$\left(\frac{40\sqrt{6}}{3}\right)^-$	$\frac{40\sqrt{6}}{3}$	$\left(\frac{40\sqrt{6}}{3}\right)^+$
$\frac{dV}{dx}$	+	0	–
sketch of tangent	/	—	\

[M1 – first or 2nd derivative, ecf 1 for method]

Since this value of r give maximum V value, the prototype satisfies the scientist's requirement as he could maximise the fuel to be stored in the prototype. [B1]

9	It is given that $f(x)=4+\cos\left(\frac{x}{2}\right)$ and $g(x)=-2\sin x$.		
	(a)	State the period and amplitude of $f(x)$.	[2]
		Period = 720° or 4π [B1] Amplitude = 1 [B1]	
	(b)	State the period and amplitude of $g(x)$.	[1]
		Period = 360° or 2π Amplitude = 2 [B1 for both]	
	(c)	Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0° \le x \le 360°$.	[3]



	f(x)	*g(x)*
Shape	cosine	negative sine
Amplitude	1	2
No. of cycle	Half	1
Shift	4	-

[B1 – for starts and ends at 'zero' for $g(x)$]

[B1 – for starts '5' and ends at '3' for $f(x)$]

[B1 – fully correct graphs]

10 **(a)** Express $\dfrac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

$$\frac{1-3x-3x^2}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

$1-3x-3x^2=A(x+1)^2+Bx(x+1)+Cx$ [M1]

When $x = -1$, $C=-1$ [M1]

When $x = 0$, $A=1$ [M1]

When $x = 1$, $B=-4$ [M1]

$$\frac{1-3x-3x^2}{x(x+1)^2}=\frac{1}{x}-\frac{4}{x+1}-\frac{1}{(x+1)^2}$$ [A1]

(b) Hence find $\displaystyle\int\frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

$$\int\frac{1-3x-3x^2}{2x(x+1)^2}\,dx=\frac{1}{2}\int\frac{1-3x-3x^2}{x(x+1)^2}\,dx$$

$=\dfrac{1}{2}\displaystyle\int\frac{1}{x}-\frac{4}{x+1}-\frac{1}{(x+1)^2}\,dx$ [M1 – take out 0.5]

$=\dfrac{1}{2}\ln x-2\ln(x+1)+\dfrac{1}{2(x+1)}+C$ [A1for each term,

must have constant C]

Accept $=\ln\sqrt{x}-\ln(x+1)^2+\dfrac{1}{2(x+1)}+C$

or $\ln\dfrac{\sqrt{x}}{(x+1)^2}+\dfrac{1}{2(x+1)}+C$

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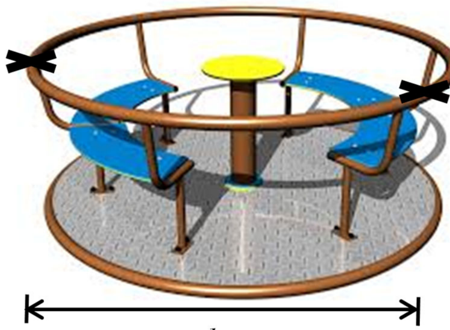


The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

(a) Explain why this model suggests that the diameter of the carousel is 4 m. [1]

$h = 2(1 - \cos kt)$.

Since the diameter of the carousel = max value of h, $h = 2\,(1 - (-1)) = 4$ m

[B1 – must relate diameter to *h, do not accept amplitude method as question ask on the model equation*]

(b) Show that the value of k is $\frac{\pi}{10}$ radians per second. [2]

Period = 20 s

$\frac{2\pi}{k} = 20$ [M1]

$k = \frac{\pi}{10} \, rad/s \quad (shown)$ [A1]

accept
2π rad in 20 s
$\frac{2\pi}{20}$ rad in 1 s
$\frac{2\pi}{20}t$ rad in t s
$k = \frac{\pi}{10}$

accept
$h = 4$ when $t = 10$
to find k value

(c) As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

$1 = 2(1 - \cos\frac{\pi}{10}t)$

accept $2(1 - \cos\frac{\pi}{10}t) > 1$

[M1]

$\frac{1}{2} = \cos\frac{\pi}{10}t$

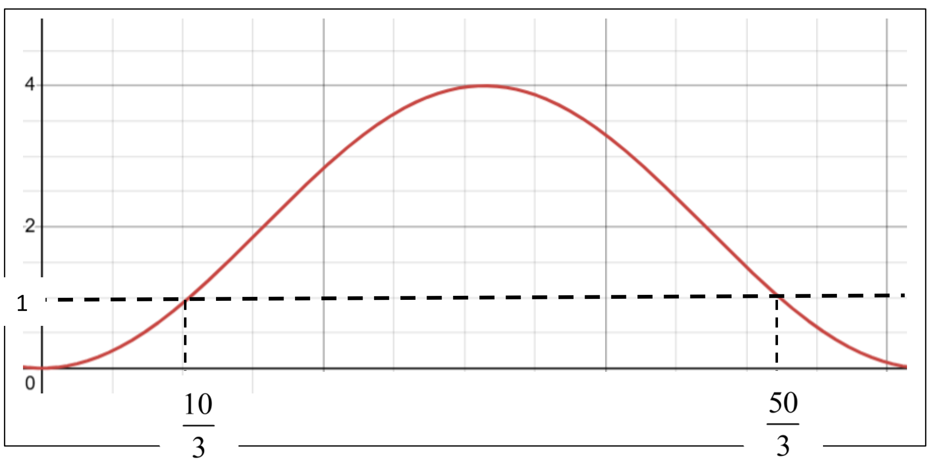
$basic\angle = \frac{\pi}{3}$ (solution in 1st and 4th quad)

$\frac{\pi}{10}t = \frac{\pi}{3}, \frac{5\pi}{3}$ [M1]

$t = \frac{10}{3}, \frac{50}{3}$ [M1]

accept $t > \frac{10}{3}$ $t < \frac{50}{3}$

Not able to view: $\frac{50}{3} - \frac{10}{3} = \frac{40}{3} s$ *or* $13.3\ s\ (3s.f.)$ [M1, A1]



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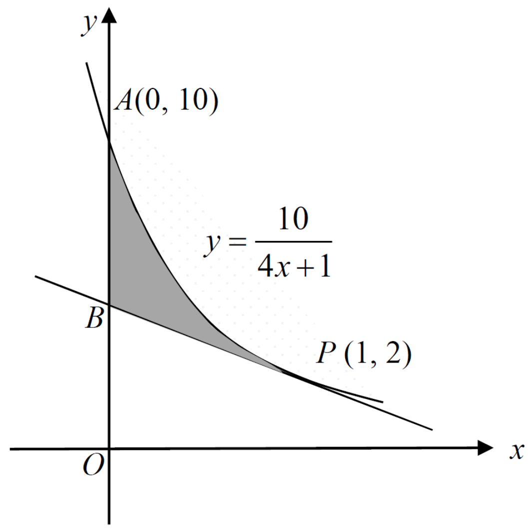

The diagram shows part of the curve $y = \frac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.

(a) Show that the coordinates of B is (0, 3.6). [4]

$y = \frac{10}{4x+1} = 10(4x+1)^{-1}$

$\frac{dy}{dx} = -10(4x+1)^{-2}(4)$ [M1]

$\frac{dy}{dx} = -40(4x+1)^{-2}$

When $x = 1$ $\frac{dy}{dx} = -40(4(1)+1)^{-2}$

$\frac{dy}{dx} = -1.6$ [M1]

$\frac{y-2}{0-1} = -1.6$ [M1]

$y - 2 = 1.6$

$y = 3.6$

Coordinate of B is (0, 3.6) [A1]

(b) Find the **exact** area of the shaded region. [5]

$Area = \int_0^1 \frac{10}{4x+1}\,dx - \frac{1}{2}(3.6+2)(1)$	[M1], [M1]
$Area = \left[\frac{10\ln(4x+1)}{4}\right]_0^1 - 2.8$	[M1]
$Area = \left[\frac{10\ln(4+1)}{4} - \frac{10\ln(1)}{4}\right] - 2.8$	[M1]
$Area = \frac{5}{2}\ln 5 - 2.8 \text{ unit }^2$	[A1]

END OF PAPER