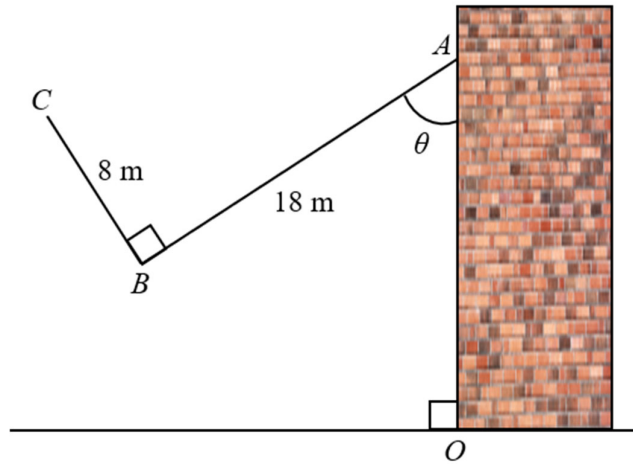


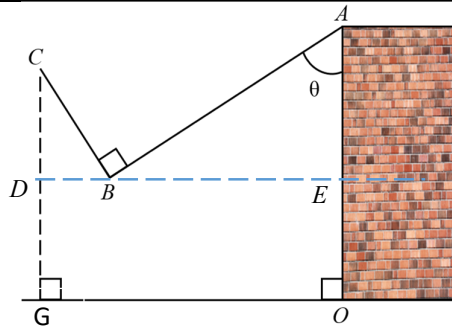
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|---|---|
| 1 | A curve has the equation $y = \frac{\sin 2x}{2 - \cos 2x}$. |
| | <p>(a) Show that the gradient function can be expressed in the form $\frac{k \cos 2x - 2}{(2 - \cos 2x)^2}$, where k is a constant. [3]</p> |
| | $y = \frac{\sin 2x}{2 - \cos 2x}$ $\frac{dy}{dx} = \frac{(2 - \cos 2x)2 \cos 2x - \sin 2x(2 \sin 2x)}{(2 - \cos 2x)^2}$ $= \frac{4 \cos 2x - 2 \cos^2 2x - 2 \sin^2 2x}{(2 - \cos 2x)^2}$ $= \frac{4 \cos 2x - 2(\cos^2 2x + \sin^2 2x)}{(2 - \cos 2x)^2}$ $= \frac{4 \cos 2x - 2}{(2 - \cos 2x)^2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;">M1: correct quotient/pdt rule</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;">M1: differentiate $\sin 2x$ and $\cos 2x$ correctly</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;">A1: use of identity to reach answer</div> |
| | <p>(b) Find the acute angle between the tangent to the curve at $x = \frac{\pi}{12}$ and the line $y = 0$. [3]</p> |
| | $\text{gradient of tangent} = \frac{4 \cos \frac{\pi}{6} - 2}{\left(2 - \cos \frac{\pi}{6}\right)^2} = 1.1386$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;">M1: correct gradient value</div> $\text{Angle required} = \tan^{-1}(1.1386) = 48.7^\circ \text{ (1 dp)}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;">M1, A1 (accepts 0.850 rad)</div> |

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| 2 | (a) Factorise $x^3 + 27k^3$ as a product of a linear and a quadratic factor. [2] |
| | $x^3 + 27k^3 = (x+3k)(x^2 - 3kx + 9k^2)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> M1: $(x+3k)$ A1 </div> |
| | (b) Hence solve $x^3 + 27 = (x+3)(x+10)$, expressing non-integer roots in surd form. [3] |
| | $k=1: x^3 + 27 = (x+3)(x^2 - 3x + 9)$ $(x+3)(x^2 - 3x + 9) = (x+3)(x+10)$ $x+3=0 \text{ or } x^2 - 3x + 9 = x+10$ $x = -3 \text{ or } x^2 - 4x - 1 = 0$ $x = \frac{4 \pm \sqrt{16 - 4(-1)}}{2}$ $= 2 \pm \sqrt{5}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> B1: identify $x=-3$ as a root from Hence M1: apply quad formula correctly A1 </div> |
| | (c) Find the value of k given that $x^3 + 27k^3$ leaves a remainder of 351 when divided by $x-2$. [2] |
| | Let $f(x) = x^3 + 27k^3$ $f(2) = 2^3 + 27k^3 = 351$ $k = \sqrt[3]{\frac{351-8}{27}} = \frac{7}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> M1: applying remainder thm correctly A1 </div> |

- 3 The diagram shows a L -shaped rod ABC where AB and BC have length 18 m and 8 m respectively and angle ABC is 90° . The rod is hinged to a wall at A so as to rotate in a vertical plane. The rod AB makes an acute angle θ with the vertical wall surface OA .



- (a) Given that G is a point directly below C , show that $OG = p \cos \theta + q \sin \theta$, where p and q are constants to be found. [2]



Using triangle BCD : Angle $DBC = \theta$

$$\cos \theta = \frac{BD}{8}$$

$$BD = 8 \cos \theta$$

B1: either correctly establishing BD or BE

Using triangle ABE : Angle $BAE = \theta$

$$\sin \theta = \frac{BE}{18}$$

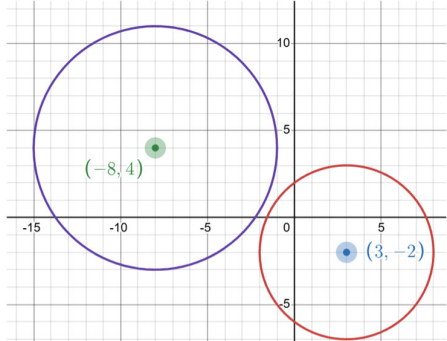
$$BE = 18 \sin \theta$$

$$\begin{aligned} OG &= BD + BE \\ &= 8 \cos \theta + 18 \sin \theta, \quad p = 8, \quad q = 18 \end{aligned}$$

B1: showing the other component and clear indication that OG is a sum of the 2 values

- (b) Express OG in the form $R \cos(\theta - \alpha)$ where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$. [3]

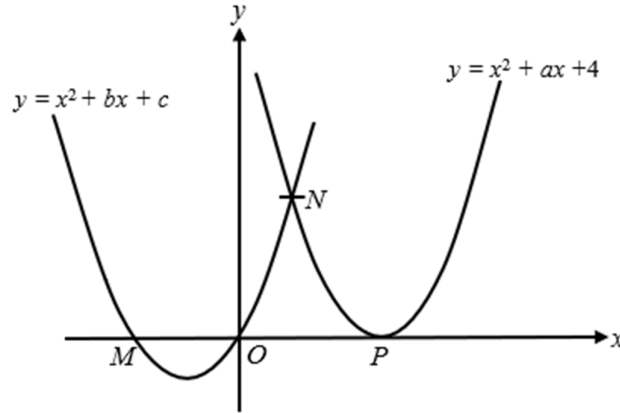
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| | $OG = 8\cos\theta + 18\sin\theta$ $= \sqrt{8^2 + 18^2} \cos\left(\theta - \tan^{-1}\frac{18}{8}\right)$ $= \sqrt{388} \cos\left(\theta - \tan^{-1}\frac{9}{4}\right)$ $= 19.7 \cos(\theta - 66.0^\circ)$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>M1: $\sqrt{8^2 + 18^2}$</p> <p>M1: $\tan^{-1}\frac{18}{8}$</p> <p>A1: correct evaluation and form, accept $\sqrt{388}$</p> </div> |
| | <p>(c) Find the length of OG and the corresponding value of θ if G is at maximum displacement from O. [3]</p> |
| | $\max OG = \sqrt{388}$ $= 19.7 \text{ m}$ <p>when $\cos\left(\theta - \tan^{-1}\frac{9}{4}\right) = 1$</p> $\theta - \tan^{-1}\frac{9}{4} = 0$ $\theta = \tan^{-1}\frac{9}{4} = 66.0^\circ$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>A1: $\sqrt{388}$ or $19.7m$</p> <p>M1: $\theta - \tan^{-1}\frac{9}{4} = 0$</p> <p>A1: 66.0°</p> </div> |
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| 4 | A circle C_1 has equation $x^2 + y^2 - 6x + 4y = 12$. |
| | (a) Find the radius and the coordinates of the centre of C_1 . [3] |
| | $x^2 - 6x + y^2 + 4y = 12$ $(x-3)^2 + (y+2)^2 = 12 + 3^2 + 2^2$ $(x-3)^2 + (y+2)^2 = 5^2$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: completing the square or using formula</div> Radius = 5 units Centre = (3, -2) <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto; text-align: center;">A1 B1</div> |
| | (b) Find the equation of the tangent to the circle at the point $P(7, -5)$. [3] |
| | $\text{Gradient of normal} = \frac{-2 - (-5)}{3 - 7} = -\frac{3}{4}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;">M1</div> $\text{Gradient of tangent} = \frac{4}{3}$ $\text{Eqn of tangent: } y + 5 = \frac{4}{3}(x - 7)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto; text-align: center;">M1 award for correct pts and their -1/m used</div> $y = \frac{4}{3}x - \frac{43}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto; text-align: center;">A1, or $3y = 4x - 43$</div> |
| | (c) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2] |
| | $\text{Distance between centres of circle} = \sqrt{(3+8)^2 + (-2-4)^2} = \sqrt{157}$ $\text{Shortest distance} = \sqrt{157} - 7 - 5 = 0.530 \text{ cm (3 s.f.)}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto; text-align: center;">M1 A1</div>  |

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| 5 | <p>(a) Prove the identity $\frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]</p> |
| $LHS = \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A}$ $= \frac{\sin A - \cos A + \sin^2 A \cos A - \sin A \cos^2 A}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">M1: correct expansion of terms</div> $= \frac{\sin A - \cos A + (1 - \cos^2 A) \cos A - \sin A (1 - \sin^2 A)}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">M1: applying identity</div> $= \frac{\sin A - \cos A + \cos A - \cos^3 A - \sin A + \sin^3 A}{\cos^3 A}$ $= \frac{\sin^3 A - \cos^3 A}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">M1: expand & simplify</div> $= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">A1: manipulation to RHS</div> $= \tan^3 A - 1$ <p>OR</p> $LHS = \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A}$ $= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">M1: applying identity</div> $= \frac{\sin^3 A + \sin A \cos^2 A + \sin^2 A \cos A - \sin^2 A \cos A - \cos^3 A - \sin A \cos^2 A}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">M1: correct expansion of terms</div> $= \frac{\sin^3 A - \cos^3 A}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">M1: simplify</div> $= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 150px;">A1: manipulation to RHS</div> $= \tan^3 A - 1$ | |

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| | <p>(b) Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2 \cos^3 A = 0$ exactly, for $-\pi \leq A \leq \pi$ radians. [4]</p> |
| | $(\sin A - \cos A)(1 + \sin A \cos A) = -2 \cos^3 A$ $\frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = -2$ $\tan^3 A - 1 = -2$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">M1: simplification to single trigo equation</div> $\tan^3 A = -1$ $\tan A = -1$ <p>Basic angle = $\frac{\pi}{4}$</p> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">M1: correct basic angle – must be acute</div> <p>Quad: Q2, Q4</p> $A = \frac{3\pi}{4}, -\frac{\pi}{4}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">A1, A1</div> |

- 6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x -axis at P . Points M and O are the x -intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP .



(a) Find the values of a , b and c .

[4]

Since origin $(0, 0)$ is on $y = x^2 + bx + c$, $c = 0$

B1

Discriminant of $y = x^2 + ax + 4 = 0$ since curve intersects x -axis once only:

$$a^2 - 4(1)(4) = 0$$

$$a = \pm 4$$

Since P is on positive x -axis, $a < 0$: $a = -4$

M1: using discriminant or the equation must be a perfect square

A1

OR

$$y = x^2 + ax + 4 = \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{4} + 4.$$

Since curve intersects x -axis once only:

$$4 - \frac{a^2}{4} = 0$$

$$a = \pm 4$$

Since P is on positive x -axis, $a < 0$: $a = -4$

M1: using discriminant or the equation must be a perfect square

A1

$$\text{Coor of } P = \left(-\frac{a}{2}, 0\right) = (2, 0)$$

$$\text{Coor of } M = (-b, 0) = (-2, 0)$$

$$b = 2$$

B1

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| | <p>(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at N. Find the coordinates of N. [2]</p> |
| | $x^2 - 4x + 4 = x^2 + 2x$ $6x = 4$ $x = \frac{2}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: equating and solving correct x (allows ecf)</div> $y = \left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) = \frac{16}{9}$ $\text{Coordinates of } N = \left(\frac{2}{3}, \frac{16}{9}\right)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1</div> |
| | <p>(c) The graph $y = px^2 + qx + r$ has its turning point at N and passes through point P. Find the values of p, q and r, where $r > 0$. [3]</p> |
| | <p>Graph with turning point at N and passes P (downward opening):</p> $y = -p\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}, p < 0$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: correct completed square form</div> <p>At $P(2, 0)$, $0 = -p\left(2 - \frac{2}{3}\right)^2 + \frac{16}{9}$</p> $p = -1$ $y = -\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}$ $= -\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \frac{16}{9}$ $= -x^2 + \frac{4}{3}x + \frac{4}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1: correct p</div> $p = -1, q = r = \frac{4}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1: correct q & r</div> <p>OR (longer method): form 3 equations with coor of N, P and either 2nd x-intercept or derivative and solve</p> $y = px^2 + qx + r$ $\frac{dy}{dx} = 2px + q$ <p>Turning point at $\left(\frac{2}{3}, \frac{16}{9}\right)$: $2p\left(\frac{2}{3}\right) + q = 0$</p> $4p = -3q \text{ ---- (1)}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: establishing 3 equations correctly</div> <p>Graph passes through P: $0 = p(2)^2 + q(2) + r$</p> $4p + 2q + r = 0$ |

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| | <p>Subst (1): $q = r$ ----- (2)</p> <p>Graph passes through N: $\frac{16}{9} = p\left(\frac{2}{3}\right)^2 + q\left(\frac{2}{3}\right) + r$</p> $4p + 6q + 9r = 16$ <p>Subst (1) and (2):</p> $-3q + 6q + 9q = 16$ $12q = 16$ $q = \frac{4}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;"> <p>A1: correct p</p> <p>A1: correct q & r</p> </div> |
| 7 | <p>A particle P, travels in a straight line, so that its displacement, s m, from O at time t seconds, is modelled by $s = \frac{1}{3}t^3 - 5t^2 - 3$.</p> |
| | <p>(a) Find the value of t when particle P return to its initial position. [2]</p> <p>Initial position when $t = 0, s = -3$</p> $\frac{1}{3}t^3 - 5t^2 - 3 = -3$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;"> <p>M1: setting $s = -3$</p> </div> $\frac{1}{3}t^3 - 5t^2 = 0$ $t^2\left(\frac{1}{3}t - 5\right) = 0$ <p>$t = 0$ or $t = 15s$</p> <p>Particle returns to initial position at $t = 15s$. A1</p> |
| | <p>(b) Find the minimum velocity of particle P. [3]</p> |
| | <p>$v = t^2 - 10t$</p> $= (t - 5)^2 - 25$ <p>Minimum velocity = -25 m/s^2</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;"> <p>M1: $v = t^2 - 10t$</p> <p>M1: completing the square</p> </div> <p>OR</p> <p>$v = t^2 - 10t$</p> <p>At stationary v,</p> $\frac{dv}{dt} = 2t - 10 = 0$ $t = 5$ $v = (5)^2 - 10(5) = -25 \text{ m/s}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;"> <p>A1</p> <p>M1: $v = t^2 - 10t$</p> <p>M1: solving for $t = 5$</p> </div> <p>$\frac{d^2v}{dt^2} = 2 > 0$, minimum v at -25 m/s</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;"> <p>A1: includes check for minimum</p> </div> |

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| | <p>(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right)$.</p> <p>Find the value of t at which the particle Q is instantaneously at rest. [2]</p> |
| | $v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right) = 0$ $e^{-\frac{t}{6}} = e^{-1}$ $t = 6s$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> M1: Setting $v = 0$ A1 </div> |
| | <p>(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]</p> $s = 24\left(\frac{e^{-\frac{t}{6}}}{-\frac{1}{6}} - te^{-1}\right) = -24\left(6e^{-\frac{t}{6}} + te^{-1}\right) + C$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> M1: correct integration (award even if without +C) </div> <p>When $t=0, s=0$: $s = -24(6) + C = 0$ $C = 144$</p> $s = 144 - 24\left(6e^{-\frac{t}{6}} + te^{-1}\right)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> M1: correct S </div> <p>When $t = 6, s = 144 - 24(6e^{-1} + 6e^{-1}) = 38.051m$ (5 sf)</p> <p>When $t = 9, s = 144 - 24\left(6e^{-\frac{9}{6}} + 9e^{-1}\right) = 32.407m$ (5 sf)</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> M1: sub $t=6$ and $t=9$ </div> <p>Total distance travelled = $s = 38.051 + (38.051 - 32.407) = 43.7m$ (3 sf)</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> A1 </div> <p>OR</p> <p>Displacement covered for 1st 6 sec =</p> $\int_0^6 24\left(e^{-\frac{t}{6}} - e^{-1}\right) dt = 24\left[\frac{e^{-\frac{t}{6}}}{-\frac{1}{6}} - te^{-1}\right]_0^6$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> M1: correct integration </div> $= -24\left[6e^{-\frac{t}{6}} + te^{-1}\right]_0^6$ $= -24\left[6e^{-\frac{6}{6}} + 6e^{-1} - 6e^{-\frac{0}{6}} - 0\right]$ $= 38.051m$ (3sf) <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: 100px;"> M1 </div> <p>Displacement covered for next 3 sec =</p> |

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| | $\int_6^9 24 \left(e^{-\frac{t}{6}} - e^{-1} \right) dt = -24 \left[6e^{-\frac{t}{6}} + te^{-1} \right]_6^9$ $= -24 \left[6e^{-\frac{9}{6}} + 9e^{-1} - 6e^{-\frac{6}{6}} - 6e^{-\frac{6}{6}} \right]$ $= -5.6434m \text{ (5sf)}$ | M1 |
| | Total distance = $s = 38.051 + (38.051 - 32.407) = 43.7m$ | A1 |

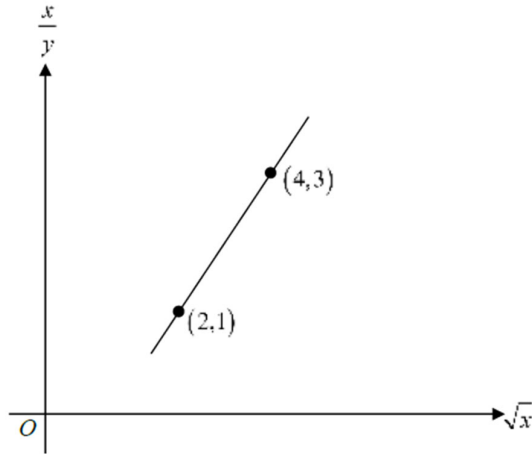
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| 8 | <p>The height, h cm, of a plant is modelled by $h = \frac{80}{1 + 10e^{-0.4t}}$, where t is the number of months after the first observation.</p> | |
| | <p>(a) Show that h is an increasing function. [3]</p> | |
| | $\frac{dh}{dt} = 80 \frac{d}{dt} (1 + 10e^{-0.4t})^{-1}$ $= 80(-1)(1 + 10e^{-0.4t})^{-2} (10(-0.4)e^{-0.4t})$ $= \frac{320e^{-0.4t}}{(1 + 10e^{-0.4t})^2}$ | M1: differentiate $1 + 10e^{-0.4t}$ correctly |
| | <p>Since $e^{-0.4t} > 0$ for $t > 0$, $320e^{-0.4t} > 0$ and $(1 + 10e^{-0.4t})^2 > 0$</p> $\frac{dh}{dt} = \frac{320e^{-0.4t}}{(1 + 10e^{-0.4t})^2} > 0$ <p>Therefore, h is an increasing function.</p> | M1: simplification |
| | <p>A1: Clear explanation on why derivative is positive and therefore h is increasing</p> | |
| | <p>(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]</p> | |
| | <p>Since h is increasing,</p> $\frac{80}{1 + 10e^{-0.4t}} > \frac{4 \times 80}{1 + 10e^0}$ $1 + 10e^{-0.4t} < \frac{11}{4}$ $e^{-0.4t} < \frac{\frac{11}{4} - 1}{10} = 0.175$ $-0.4t < \ln 0.175$ $t > 4.3574 \text{ (5 s.f.)}$ $t = 4.36 \text{ (3 s.f.)}$ | M1: correct set up of equation (accepts equal if students round up at the end, preferably with explanation that h is an increasing function) |
| | | M1: simplification to exponential eqn |
| | <p>A1 (accepts 5, though context need not be integer)</p> | |

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| | <p>(c) The height, y cm, of another species of plant, t months after the first observation is given by $y = \frac{1}{a + be^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]</p> |
| | $y = \frac{1}{a + be^{-t}}$ $\frac{1}{y} = a + be^{-t}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: simplification to $Y=mX + c$ form</div> <p>Plot $\frac{1}{y}$ on the vertical axis against e^{-t} on the horizontal axis to obtain a straight line graph. [A1] The gradient of the graph would give the value of b [A1] and the y-intercept will give the value of a [A1].</p> <p>Note: no award of full A1s if M1 not awarded.</p> |
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| 9 | <p>(a) Without using a calculator, solve the equation $x\sqrt{15} + \sqrt{5} = x\sqrt{2} + \sqrt{6}$. Leave your answer in the form $p\sqrt{10} + q\sqrt{3}$, where p and q are fractions. [4]</p> |
| | $x\sqrt{15} + \sqrt{5} = x\sqrt{2} + \sqrt{6}$ $x(\sqrt{15} - \sqrt{2}) = \sqrt{6} - \sqrt{5}$ $x = \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: $x = \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}}$</div> $= \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}} \times \frac{\sqrt{15} + \sqrt{2}}{\sqrt{15} + \sqrt{2}}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: correct rationalisation fraction</div> $= \frac{\sqrt{90} - \sqrt{10} - \sqrt{75} + \sqrt{12}}{15 - 2}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">M1: expansion of numerator and denominator</div> $= \frac{3\sqrt{10} - \sqrt{10} - 5\sqrt{3} + 2\sqrt{3}}{13}$ $= \frac{2\sqrt{10} - 3\sqrt{3}}{13}$ $= \frac{2}{13}\sqrt{10} - \frac{3}{13}\sqrt{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">A1: in $p\sqrt{10} + q\sqrt{3}$, where p and q are fractions</div> |

| | |
|--|--|
| | <p>(b) Without using a calculator, solve the equation $\log_2 x - \log_x 16 = 0$. [4]</p> <p>$\log_2 x - \log_x 16 = 0$</p> <p>$\log_2 x - \frac{\log_2 16}{\log_2 x} = 0$ M1: correct change of base</p> <p>$(\log_2 x)^2 = 4$ M1: simplification of $\log_2 16$ and reduced to a solvable quadratic equation</p> <p>$\log_2 x = \pm 2$</p> <p>$x = 2^2$ or $x = 2^{-2}$</p> <p>$x = 4$ or $x = \frac{1}{4}$ A1, A1</p> |
| | <p>(c) Solve the equation $3^{x+2} - 2(3^{-x}) = 17$. [4]</p> <p>$3^{x+2} - 2(3^{-x}) = 17$</p> <p>$9(3^x) - \frac{2}{3^x} = 17$</p> <p>$9(3^x)^2 - 17(3^x) - 2 = 0$ M1: correct application of indices rules and rewriting into quadratic eqn in 3^x</p> <p>Let $u = 3^x$</p> <p>$9u^2 - 17u - 2 = 0$</p> <p>$(u - 2)(9u + 1) = 0$ M1: factoring or formula to solve quadratic eqn</p> <p>$u = 2$ or $u = -\frac{1}{9}$ (rej since $3^x > 0$) A1: Solving of 3^x with rejection</p> <p>$3^x = 2$</p> <p>$x = \frac{\ln 2}{\ln 3} = 0.631$ (3 s.f.) A1: do not accept $x = \log_3 2$ as no way of computing</p> |

- 10 (a) The diagram shows part of a straight line graph which passes through (2,1) and (4,3).



Find the equation of the straight line in the form $y = \frac{x}{a+b\sqrt{x}}$, where a and b are constants. [3]

gradient of straight line: $\frac{3-1}{4-2} = 1$

equation:

$$Y - 1 = 1(X - 2)$$

$$Y = X - 1$$

$$\frac{x}{y} = \sqrt{x} - 1$$

$$y = \frac{x}{\sqrt{x} - 1}$$

M1: gradient of line

M1: eqn of line

A1

- (b) The table below shows the experimental values of two variables x and y .

| | | | | | | |
|-----|----|-----|-----|-----|------|------|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 63 | 127 | 258 | 510 | 1000 | 2100 |

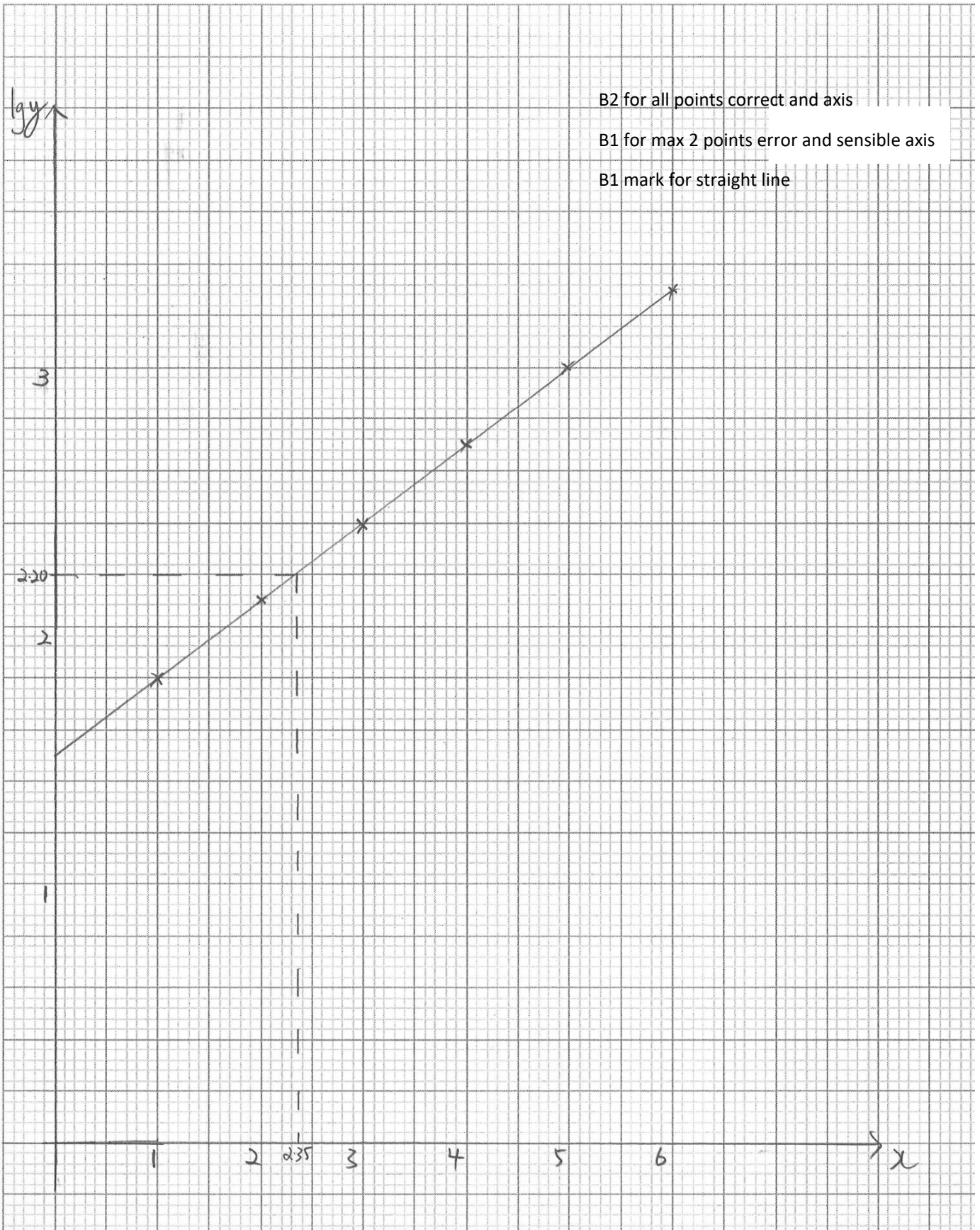
It is known that x and y are related by an equation of the form $y = \frac{b^x}{10^a}$, where a and b are constants.

- (i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

- (ii) Use your graph to estimate the value of a and of b . [3]

$$y = \frac{b^x}{10^a} \Rightarrow$$

| | |
|--|--|
| | $\lg y = \lg \left(\frac{b^x}{10^a} \right)$ $\lg y = \lg b^x - \lg 10^a$ $\lg y = x \lg b - a \quad \text{M1}$ $-a = 1.5$ $a = -1.5 \quad \text{A1 (accept -1.54 to -1.46)}$ $\lg b = 0.3$ $b = 2.00 \quad \text{A1 (accept 1.82 to 2.19)}$ |
| | <p>(iii) Explain how you would use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]</p> |
| | $(10b)^x = 10^{a+1}$ $10^x b^x = 10^a \times 10$ $\frac{b^x}{10^a} = \frac{10}{10^x} \quad \text{M1}$ $y = 10^{1-x}$ $\lg y = \lg(10^{1-x})$ $\lg y = 1 - x$ <p>Draw the line $\lg y = 1 - x$ and find the x-coordinate of the point of intersection. A1</p> <p>OR</p> $(10b)^x = 10^{a+1}$ $x \lg(10b) = (a+1) \lg(10)$ $x \lg(10) + x \lg(b) = a+1$ $x \lg(b) - a = x+1$ $\lg y = x+1$ |



B2 for all points correct and axis

B1 for max 2 points error and sensible axis

B1 mark for straight line